Thomson's New Mathematical Series

A

COMPLETE

Intellectual Arithmetic

THOMSON

COMPLETE

INTELLECTUAL ARITHMETIC;

ADAPTED TO CLASSES IN

GRAMMAR SCHOOLS AND ACADEMIES.

BY

JAMES B. THOMSON, LL. D.,

AUTHOR OF NEW MATHEMATICAL SERIES.

NEW YORK:

MAYNARD, MERRILL, & CO.,

29, 31, AND 33 EAST NINETEENTH STREET.

THOMSON'S MATHEMATICAL SERIES.

I. *A Graded Series of Arithmetics, in three Books, viz.:*

New Illustrated Table Book, or Juvenile Arithmetic. With oral and slate exercises. (For beginners.) 128 pp.

New Rudiments of Arithmetic. Combining Mental with Written Arithmetic. (For Intermediate Classes.) 224 pp.

New Practical Arithmetic. Adapted to a complete business education. (For Grammar Departments.) 384 pp.

II. *Independent Books.*

Key to New Practical Arithmetic. Containing many valuable suggestions. (For teachers only.) 168 pp.

New Mental Arithmetic. Containing the Simple and Compound Tables. (For Primary Schools.) 144 pp.

Complete Intellectual Arithmetic. Specially adapted to Classes in Grammar Schools and Academies. 168 pp.

III. *Supplementary Course.*

New Practical Algebra. Adapted to High Schools and Academies. 312 pp.

Key to New Practical Algebra. With full solutions. (For teachers only.) 224 pp.

New Collegiate Algebra.

Complete Higher Arithmetic. (In preparation.)

*** *Each book of the Series is complete in itself.*

PREFACE.

INTELLECTUAL ARITHMETIC has, confessedly, never received that attention in popular education which its importance demands. The distaste for the study, often manifested, is due not so much to the subject itself, as to the manner in which it is presented.

Mental Arithmetic is but another name for Arithmetical Analysis, which is the master-key to the treasury of Mathematics. It unlocks the portals and bids the tyro enter and participate freely in the riches of this matchless science.

By the aid of Analysis, the business man performs his multifarious calculations with readiness and accuracy, though the rules of the book have long since faded from his memory; by the application of its principles the scholar threads the intricate mazes of Astronomical Science.

The plan of this book is to present a series of mental exercises adapted to classes in Grammar Schools and Academies. It embraces all the more important principles of business arithmetic, without being encumbered with operations in counting the fingers, or with unwieldy combinations of numbers.

The problems cover a wide field of subjects and principles, both practical and scientific. These subjects and

principles are classified, and arranged under appropriate heads in the order of their dependence, a feature which contributes largely to the convenience and value of a text-book for use in the class-room.

Special attention has been paid to the *Methods* of Analysis. The first example involving a new principle is analyzed, care being taken to make the steps consecutive, distinct, clear, and logical.

In accordance with the demands of the age, the Metric System of Weights and Measures is introduced and fully explained. Its primary units are illustrated by diagrams, and the meaning of the principal denominations is made familiar, by a series of mental exercises comparing their approximate values with the values of the weights and measures in common use.

The introduction of this system is a new feature in Mental Arithmetic, and it is believed will be regarded by the friends of progress as a step in the right direction.

The work, it is hoped, will meet the wants of teachers in this department, and be instrumental in awakening a deeper interest in this important branch of popular education.

When its principles are thoroughly mastered, the pupil will be in possession of a "Ready Reckoner," always at hand, always reliable, and of universal application.

J. B. THOMSON.

BROOKLYN, July, 1878.

CONTENTS.

INTELLECTUAL ARITHMETIC.

CHAPTER I.

ADDITION.

Art. 1. Ex. 1. A man picked 8 pears from one tree, and 7 pears from another: how many pears did he pick from both trees?

Analysis.—8 pears and 7 pears are 15 pears. Therefore, he picked 15 pears.

Or, briefly, 8 and 7 are 15. *Ans.* 15 pears.*

2. If you solve 6 examples on Monday and 9 on Tuesday, how many will you solve in 2 days?

3. Howard had 7 quarts of chestnuts and George 9 quarts: how many quarts had both?

4. One holiday James caught 3 fish, John 9, and Thomas 5: how many did all catch?

5. How many are 5 and 8? 8 and 7? 6 and 9?

6. How many are 8 and 6? 6 and 7? 8 and 9?

7. How many are 4, 5 and 7? 6, 3 and 8?

8. How many are 5, 9 and 4? 7, 5 and 6?

9. How many are 8, 3 and 9? 8, 7 and 9?

10. If a man walks 7 miles in the morning and 8 in the afternoon, how many miles will he walk in a day?

11. A lady paid $9 for a dress, $8 for a hat, and $5 for a collar: what did she pay for all?

* The pupil is supposed to be familiar with the simple Tables; if not, portions of them should be reviewed daily till they are thoroughly mastered.

12. What is the process called by which the preceding examples are solved?

13. How is Addition denoted?

Ans. By ***a perpendicular cross*** called ***plus;**** as, +.

Thus, 6 + 4 means that 4 is to be added to 6, and is read "6 plus 4."

14. How is equality denoted?

Ans. By ***two short horizontal lines, equal*** and ***parallel;*** as, =.

Thus, 4 + 3 = 7, shows that 4 and 3 equal 7.

15. 7 + 9 + 8 = how many?
16. 6 + 8 + 7 = how many?
17. 25 + 6 + 7 = how many?
18. 46 + 9 + 8 = how many?

2. Ex. 1. If a slate costs 19 cents, and a sponge 14 cents, what will both cost?

Analysis.—19 cents are 5 more than 14 cents. Now 14 cents and 14 cents are 28 cents, and 5 are 33 cents. Therefore, both will cost 33 cents.

Or, briefly, 19 is 5 more than 14. But 14 and 14 are 28, and 5 are 33. *Ans.* 33 cents.

2. A man paid $16 for a saddle, and $15 for a bridle: how much did he pay for both?

3. If you gather 14 peaches from one tree, and 16 from another, how many peaches will you have?

4. Suppose a lad walks 16 miles one day, and 17 the next, how far will he walk in both days?

5. After losing 17 marbles, George had 19 left: how many marbles had he at first?

6. How many are 17 and 13? 16 and 18? 15 and 19? 15 and 16? 19 and 18?

* The term *plus*, signifies *more*, or *added* to.

7. If you pay $19 for a coat, and $17 for a hat and vest, what will be the amount of your bill?

8. William is now 14 years old: how old will he be 18 years hence?

9. Sanford was 16 years old 13 years ago: how old is he now?

10. If a drum costs $13 and a flute $18, what will both cost?

11. Harry's age is 14 years and Charles is 18: what is the sum of their ages?

12. A farmer sold a cow for $27 and a ton of hay for $27: what did he receive for both?

13. If a lad earns 25 cents in the morning and 29 cents in the afternoon, how much will he earn in a day?

14. If you pay 34 cents for an arithmetic and 40 cents for a reader, what will both cost you?

15. If a music box costs $40 and a cow $42, what will both cost?

16. If a train of cars goes 45 miles the first hour and 35 miles the second, how far will it go in two hours?

3. Ex. 1. A certain farmer had 30 cows, and his neighbor had 40: how many cows had both?

Analysis.—30 is the same as 3 tens, and 40 the same as 4 tens: now 3 tens and 4 tens are 7 tens or 70. Therefore, both have 70 cows.

Or, briefly, 3 tens and 4 tens are 7 tens or 70. *Ans.* 70 cows.

2. If there are 20 peach trees and 50 pear trees in an orchard, how many trees does the orchard contain?

3. James has 30 chestnuts and Harry 50: how many have both?

4. To what are 6 tens equal? 8 tens? 9 tens? 12 tens? 14 tens? 11 tens? 13 tens? 15 tens? 17 tens? 16 tens? 18 tens? 19 tens? 20 tens?

5. How many are 5 tens and 8 tens? 6 tens and 9 tens? 7 tens and 8 tens? 8 tens and 9 tens?

6. In a certain school there are 60 boys and 70 girls: how many scholars does the school contain?

7. If a man should pay $90 for his horse, and $70 for his wagon, how much would they both cost?

8. A drover had 80 sheep and 90 lambs: how many animals had he in his drove?

9. A farmer paid $80 for a yoke of oxen and $60 for a cart: how much did he pay for both?

10. A miller received 70 bushels of wheat of one man, 20 bushels of another, and 30 bushels of another: how many bushels did he receive from all?

11. What is the sum of 50, and 40, and 60?

12. What is the sum of 70 + 50 + 20?

13. What is the sum of 60 + 80 + 30?

14. A man paid $50 for a sleigh, $40 for a cart, and $30 for a wagon: how much did he pay for all?

15. What is the sum of 80 + 70 + 20 + 7?

16. What is the sum of 90 + 80 + 30 + 9 + 6?

4. Ex. 1. A man paid $75 for a horse, and $58 for a wagon: how much did he pay for both?

Analysis.—75 = 7 tens and 5 units, and 58 = 5 tens and 8 units; now 7 tens and 5 tens are 12 tens, or 120; 5 units and 8 units are 13 units, which added to 120, make 133. Therefore, etc.

Or, thus: 75 = 50 plus 25, and 58 = 50 plus 8; now 50 and 50 are 100, and 25 and 8 are 33, which added to 100 make 133. *Ans.* $133 for both. (Art. 2.)

NOTE.—In adding large numbers mentally, it is more *convenient* and *expeditious* to begin with the *highest* order.

2. A farmer had two lots, one of which contained 47 acres, and the other 38 acres: how many acres were there in both?

3. A young man having spent $37, found he had $56 left: how much money had he at first?

4. A farmer paid a man 87 cents for a day's work, and a boy 56 cents: how much did he pay both?

5. If Henry reads 64 pages one day and 57 the next day, how many pages will he read in both days?

6. If a man is 45 years old and his wife 39 years, what is the sum of their ages?

7. The Sandwich Islands are 20 degrees north latitude, and the Society Islands 18 degrees south latitude: how many degrees apart are they?

8. The mouth of the Columbia River is 46 north latitude, and Cape Horn 56 south Latitude: how far apart are they?

9. How many are $37 + 6 + 9 + 4$?

10. How many are $48 + 9 + 7 + 5 + 6$?

11. How many are $59 + 6 + 8 + 3 + 7 + 2$?

5. Exercises in Adding Digits to Decades.

Ex. 1. 9 and 4 are how many? 19 and 4? 29 and 4? 39 and 4? 49 and 4? 59 and 4? 69 and 4? 79 and 4? 89 and 4? 99 and 4?

2. 8 and 5 are how many? 18 and 5? 28 and 5? 38 and 5? 48 and 5? 58 and 5? 68 and 5? 78 and 5? 88 and 5? 98 and 5?

3. 7 and 7 are how many? 17 and 7? 27 and 7? 47 and 7? 37 and 7? 67 and 7? 57 and 7? 87 and 7? 77 and 7? 97 and 7?

4. 8 and 4 are how many? 18 and 4? 28 and 4? 38 and 4? 58 and 4? 48 and 4? 78 and 4? 68 and 4? 88 and 4? 98 and 4?

5. 9 and 5 are how many? 19 and 5? 39 and 5? 29 and 5? 49 and 5? 69 and 5? 59 and 5? 99 and 5? 89 and 5? 79 and 5?

6. 6 and 8 are how many? 16 and 8? 46 and 8? 36 and 8? 26 and 8? 56 and 8? 76 and 8? 66 and 8? 86 and 8? 96 and 8?

7. 6 and 9 are how many? 15 and 9? 35 and 9? 25 and 9? 45 and 9? 65 and 9? 55 and 9? 75 and 9? 85 and 9? 95 and 9?

8. 9 and 7 are how many? 16 and 7? 36 and 7? 26 and 7? 46 and 7? 66 and 7? 56 and 7? 86 and 7? 76 and 7? 96 and 7?

9. 7 and 8 are how many? 15 and 8? 25 and 8? 45 and 8? 35 and 8? 55 and 8? 75 and 8? 65 and 8? 85 and 8? 95 and 8?

10. 8 and 4 are how many? 18 and 4? 48 and 4? 38 and 4? 68 and 4? 58 and 4? 88 and 4? 78 and 4? 98 and 4?

11. 7 and 6 are how many? 17 and 6? 37 and 6? 27 and 6? 57 and 6? 47 and 6? 67 and 6? 97 and 6? 77 and 6? 87 and 6?

12. 9 and 8 are how many? 17 and 8? 27 and 8? 47 and 8? 37 and 8? 57 and 8? 77 and 8? 67 and 8? 87 and 8? 97 and 8?

13. 5 and 10? 25 and 10? 15 and 10? 75 and 10? 65 and ten? 55 and 10? 45 and 10? 95 and 10? 85 and 10?

14. 6 and 10? 8 and 10? 26 and 10? 27 and 10? 17 and 10? 68 and 10? 36 and 10? 57 and 10? 76 and 10? 88 and 10?

15. 7 and 10? 29 and 10? 47 and 10? 36 and 10? 97 and 10? 76 and 10? 69 and 10? 77 and 10? 87 and 10?

16. 9 and 10? 18 and 10? 29 and 10? 38 and 10? 69 and 10? 58 and 10? 79 and 10? 49 and 10? 89 and 10? 99 and 10?

6. Drill Exercises in Adding.

NOTE.—The following combinations include all the results that can be obtained by adding *any two digits* together; or by adding *any digit* to a *significant* figure denoting the excess of *even tens* in any decade less than a *hundred*. When these are *mastered*, the *process of adding* is mastered.

They may be recited by the class *individually* or in *concert*; or what is better, by the *alternation* of the two methods.

Ex. 1. Add 2 to itself continually, till the sum is 100.

SUGGESTION.—Omitting the "and," the pupil should simply pronounce the successive results; as, two, four, six, eight, ten, etc.

2. Add 3 to itself continually, till the sum is 102.
3. Add 4 to itself continually, till the sum is 100.
4. Add 5 to itself continually, till the sum is 100.
5. Add 6 to itself continually, till the sum is 102.
6. Add 7 to itself continually, till the sum is 105.
7. Add 8 to itself continually, till the sum is 104.
8. Add 9 to itself continually, till the sum is 108.
9. Add 2 to 1 continually, till the sum is 100.

Thus, *Three, five, seven, nine, eleven, thirteen*, etc.

10. Add 3 to 1 continually, till the sum is 100.
11. Add 3 to 2 continually, till the sum is 100.
12. Add each of the other 9 digits to 2 in a similar manner, till the sum reaches 100.
13. Add each of the digits to 3, till the sum reaches 100.
14. Add each of the digits to 4, till the sum reaches 100.
15. Add each of the digits to 5, till the sum reaches 100.
16. Add each of the digits to 6, till the sum reaches 100.
17. Add each of the digits to 7, till the sum reaches 100.
18. Add each of the digits to 8, till the sum reaches 100.
19. Add each of the digits to 9, till the sum reaches 100.

NOTE.—The practice of *subtracting* each of the digits from 100 continually, till the remainder is less than the digit, is a valuable exercise.

CHAPTER II.

SUBTRACTION.

7. Ex. 1. A lad paid 15 cents for a slate and 9 cents for a writing-book: how much more did the slate cost than the writing-book?

Analysis.—Since 9 and 6 are 15, it follows 9 cents from 15 cents leave 6 cents. Hence, the slate cost 6 cents more than the writing-book.

Or, thus: 9 from 15 leaves 6. *Ans.* 6 cents.

2. If you have $17 and spend $8, how many dollars will you have left?

3. A farmer having 16 cows, sold 7 of them: how many did he have left?

4. A hunter saw 15 crows upon a fence, and killed 4 of them at a shot, how many escaped?

5. A young man is 20 years old to day: how old was he 6 years ago?

6. How many are 13 less 5?

7. How many are 15 less 6?

8. How many are 17 less 8?

9. Three from 14 leaves how many? 5 from 17?

10. Four from 15 leaves how many? 6 from 13? 7 from 16?

11. Five from 19 leaves how many? 9 from 17? 8 from 14? 6 from 19?

12. A lad bought a pair of skates for 27 shillings, and sold them at a loss of 5 shillings: how much did he get for them?

13. A man being asked how much money he had in his purse, said if he had $13 more he would have a double eagle: how much had he?

14. What number must be added to 19 to make 25? To make 23? 26? 28? 30?

15. If a man has 16 acres of land, how many acres must he obtain to have 21 acres?

16. Five and what number make 12? 13 and what make 18? 8 and what make 14? 4 and what make 19?

17. Twelve is how many more than 4? 16 than 7? 18 than 9? 20 than 14? 29 than 15?

18. Thirteen is how many less than 18? 12 than 17? 15 than 19? 16 than 21? 24 than 30?

19. In 9 years Horace will be 17 years old: what is his present age?

Analysis.—Since in 9 years he will be 17, his present age must be 17 years less 9 years; and 9 from 17 leaves 8. Therefore, he is now 8 years old.

20. The cost of a saddle is $14, which is $8 more than the cost of a bridle: what is the cost of a bridle?

21. A lad being asked how many marbles he had, replied if he had 9 marbles more, he would then have 26: how many had he?

22. What is the process called by which the preceding examples are solved?

23. How many numbers are given in Subtraction, and what called? When we say that 4 from 9 leaves 5, which is the Minuend? The Subtrahend? The Remainder?

24. How is Subtraction denoted?

Ans. By *a short, horizontal line* called *minus;** as, —.

Thus, 6 — 4 means that 4 is to be taken from 6, and is read, "6 minus 4."

25. 18 — 3 + 5 = how many?

26. 20 — 8 — 4 = how many?

27. 35 + 7 — 6 = how many?

* The term *minus*, signifies *less*, or *diminished* by.

8. Ex. 1. If George has 80 cents in his pocket and should lose 30 of them, how many cents would he have left?

Analysis.—80 is the same as 8 tens; and 30 is the same as 3 tens; now 3 tens from 8 tens, leave 5 tens or 50. Therefore, he would have $50 left.

Or, thus: 30 from 80 leaves 50. *Ans.* $50.

2. A farmer having 70 sheep, sold 40 of them: how many did he then have?

3. A man being 90 miles from home, rode, in returning, 50 miles the first day: how far from home was he then?

4. A traveller with $80 in his pocket, was robbed of all but $20: how many dollars did he lose?

5. 20 from 60 leaves how many? 30 from 50? 40 from 80? 60 from 90? 40 from 70? 70 from 90? 60 from 80?

6. A boy having 75 cents, spent 32 cents for toys: how many cents did he then have?

Analysis.—75 is equal to 7 tens and 5 units; and 32 is equal to 3 tens and 2 units; now 3 tens from 7 tens leave 4 tens or 40, and 2 units from 5 units, leave 3 units, which added to 40 make 43. Therefore, etc.

NOTE.—When the numbers in subtraction are large, it is advisable, in mental operations, to begin at the highest orders, as in addition. (Art. 4, Note.)

7. 25 from 37 leaves how many? 42 from 65? 34 from 76? 52 from 88? 26 from 77? 75 from 98?

8. What number increased by 25 will become 48?

9. What number increased by 23 will become 54?

10. What number increased by 18 will become 79?

11. A sheepfold containing 78 sheep, was attacked by wolves, which killed 35 sheep: how many were left?

12. If you draw 31 gallons from a hogshead of molasses, how many gallons will be left?

13. Two lads together weighed 95 pounds, and one weighed 42 pounds: what was the weight of the other?

9. Ex. 1. A man gave $85 for a horse, and $47 for a wagon: how much more did he pay for his horse than for his wagon?

Analysis.—85 equals 7 tens and 15 units, and 47 equals 4 tens and 7 units; now 4 tens from 7 tens leave 3 tens or 30, and 7 units from 15 units leave 8 units, which added to 30 make 38. Therefore, etc.

NOTE.—In the analysis we transferred one of the 8 tens to the 5 units, because the units figure in the *smaller* number is greater than the units figure in the *larger* number.

2. A man having 74 acres of land, sold 56 acres: how many acres had he left?

3. 30 and what number make 43? 40 and what make 58? 60 and what make 81? 70 and what make 94? 87 and what make 100?

4. A grocer bought a hogshead of molasses, and carrying it home 36 gallons leaked out: how many gallons remained?

5. If you take 46 from 64, how many will remain? 38 from 51? 52 from 80? 66 from 72? 54 from 86? 67 from 95? 78 from 100?

6. If a farmer has 47 sheep, how many must he purchase that his flock may contain 84?

7. What number must be taken from 63 to leave 28?

8. What number must be taken from 93 to leave 76?

9. 27 and what number make 56? 45 and what make 61? 37 and what make 60? 55 and what make 71? 73 and what make 100?

10. If a man has 160 acres of land, and sells 120 acres, how many acres will he have left?

Analysis.—160 is the same as 16 tens, and 120 the same as 12 tens; now 12 tens from 16 tens leave 4 tens or 40. Therefore, etc.

11. A flock of 180 pigeons lit upon a wheat field, the owner of which sprung a net upon them, and caught 130: how many escaped?

12. 14 tens from 17 tens leave how many? 13 tens from 18 tens? 12 tens from 18 tens? 15 tens from 19 tens? 16 tens from 21 tens?

13. A drover having 300 sheep, sold 180 of them: how many did he have left?

14. A market-man having 45 dozen of eggs, sold 30 dozen: how many dozen did he have left?

15. A certain school contains 760 boys and 540 girls: how many more boys are there than girls?

10. Exercises in Subtracting Digits from Decades.

1. 12 less 3 are how many? 22 less 3? 32 less 3? 42 less 3? 52 less 3? 62 less 3? 72 less 3? 82 less 3? 92 less 3?

2. 13 less 5 are how many? 23 less 5? 33 less 5? 43 less 5? 53 less 5? 73 less 5? 93 less 5?

3. 12 less 6 are how many? 22 less 6? 32 less 6? 52 less 6? 42 less 6? 62 less 6? 72 less 6? 82 less 6? 92 less 6?

4. 14 less 7 are how many? 24 less 7? 44 less 7? 34 less 7? 54 less 7? 64 less 7? 84 less 7? 74 less 7? 94 less 7?

5. 12 less 4 leaves how many? 22 less 4? 42 less 4? 32 less 4? 52 less 4? 72 less 4? 82 less 4? 62 less 4? 92 less 4?

6. 14 less 5 are how many? 24 less 5? 44 less 5? 64 less 5? 34 less 5? 54 less 5? 74 less 5? 84 less 5? 94 less 5?

7. 16 less 8 are how many? 26 less 8? 46 less 8? 36 less 8? 66 less 8? 56 less 8? 76 less 8? 96 less 8? 86 less 8?

8. 15 less 9 are how many? 25 less 9? 45 less 9? 35 less 9? 55 less 9? 75 less 9? 65 less 9? 85 less 9? 95 less 9?

9. 16 less 7 are how many? 26 less 7? 46 less 7? 36 less 7? 56 less 7? 76 less 7? 86 less 7? 66 less 7? 96 less 7?

10. 15 less 8 are how many? 25 less 8? 45 less 8? 35 less 8? 55 less 8? 65 less 8? 85 less 8? 75 less 8? 95 less 8?

11. 12 less 4 are how many? 32 less 4? 22 less 4? 52 less 4? 42 less 4? 62 less 4? 82 less 4? 72 less 4? 92 less 4?

12. 23 less 6 are how many? 33 less 6? 53 less 6? 73 less 6? 63 less 6? 83 less 6? 93 less 6?

13. 17 less 8 are how many? 27 less 8? 47 less 8? 37 less 8? 57 less 8? 87 less 8? 77 less 8? 67 less 8? 97 less 8?

11. Drill Exercises in Adding and Subtracting.

Ex. 1. To 6 add 9; subtract 4; add 7; subtract 6; subtract 2; add 7; subtract 8; add 3: what is the result?

NOTE.—While the teacher dictates the example, "to 6 add 9, subtract 4," etc., the pupils think 15, 11, 18, 12, 10, etc.

The result may be given by the class in concert; or by some individual designated by the teacher.

2. From 14 take 4; add 8; take 7; add 6; take 3; add 5; take 19; add 10: result?

3. Add 16 to 7; subtract 5; add 4; subtract 8; add 3; add 5; subtract 6; add 2; subtract 9; add 6: result?

4. How many are 26 plus 7; minus 5; plus 8; minus 10; plus 7; minus 4; plus 7?

5. How many are 42 minus 7; plus 9; minus 4; plus 3; minus 7; plus 8; minus 10; plus 9?

6. How many are 37 minus 8; plus 7; minus 4; plus 6; minus 8; plus 12; minus 10; plus 5?

7. How many are $8 + 9 - 12 + 4 - 6 + 8 - 7 + 10$?

8. How many are $15 - 6 + 8 - 4 + 3 - 10 + 20 - 5$?

9. How many are $27+8-9+10-7+6-8+9+7+5$?

10. How many are $19+11-6+8-9+7-5+8-10$?

11. How many are $38-5+7+3-6+8-7+9-4+6$?

12. How many are $43+7-8+4-6+7+8-7-5+12$?

13. To 12 add 7, subtract 5, add 4, add 8: result?

14. From 25 take 10, add 7, add 8, take 9: result?

15. To 17 add 18, subtract 7, add 9, subtract 5: result?

16. To 26 add 18, subtract 5, add 6, subtract 9, add 5, take 8: result?

17. From 47 take 38, add 6, take 8, add 13, take 5, add 4: result?

18. Add 7 to 8, subtract 3, add 8, add 12, subtract 7, subtract 5, add 6, add 4, subtract 10: result?

19. Take 7 from 15, add 9, take 5, add 10, take 3, add 4, add 6, take 7, add 8, take 9: result?

20. From 28 take 9, add 7, take 4, take 3, add 5, add 6, take 7, add 8, add 5, take 10: result?

21. Take 12 from 48, add 6, take 7, add 8, take 3, add 7, take 9, add 11, take 4, add 3, take 5, add 9, take 10, add 7, take 8, add 9, add 3, take 5: result?

22. From 57 take 8, add 4, take 5, add 6, take 8, add 20, take 3, add 2, take 10, add 7, take 9, add 8, take 20, add 30, take 40, add 50, take 60: result?

23. To 31 add 12, subtract 10, add 6, subtract 7, add 8, subtract 4, add 9, subtract 3, add 6, subtract 8, add 10, subtract 5, add 8, subtract 2, add 6, subtract 7, add 4, subtract 6: result?

24. From 63 subtract 7, add 3, subtract 6, add 11, subtract 4, add 10, subtract 5, add 6, subtract 7, add 3, subtract 6, add 9, subtract 8, add 6, subtract 4, add 3, subtract 2, add 7: result.

25. Add 7 to 9, subtract 8, add 20, subtract 14, add 30, subtract 4, add 5, subtract 6, add 7, subtract 8, add 9, subtract 10, add 4, subtract 5, add 8, subtract 7, add 4, subtract 5: result?

QUESTIONS FOR REVIEW.

12. Ex. 1. John having 12 apples bought 6 more and sold 8: how many had he left?

2. A lad bought 6 pond lilies for 18 cts., and sold them for 11 cts.: what did he lose by his bargain?

3. A man bought an accordeon for $13, and sold it for $21: what did he gain by the bargain?

4. Two boys played at marbles; when they began they had 12 apiece, and when they had done the older had won 7: how many had each then?

5. A man bought 8 bbls. of flour for $35, and sold them at a loss of $8: how much did he get for his flour?

6. How many are 55 and 6 and 3 and 7 and 2 less 8?

7. William had 36 apples; he gave 5 to one companion, 8 to another, and when he had given some to a third, he had 6 left: how many did he give to the last?

8. Bought a watch for $12 and having paid $3 for repairs, sold it for $23: what was gained?

9. Henry having a certain number of oranges, bought 7 and sold 10; he then had none left: how many had he at first?

10. A lad having a certain number of cents, spent 12 and giving 30 to his sister, had none left: how many had he at first?

11. How many are 33 plus 9 less 12 plus 4?

12. How many are 47 plus 20 less 8 plus 6?

13. How many are $56 + 7 + 9 - 10 + 5$?

14. How many are $78 + 6 + 7 - 12 + 8$?

15. What number added to $25 - 17$ will make 14?

16. What number added to $15 + 18$ will make $65 - 11$?

17. What number subtracted from $23 + 27$ will leave 38 minus 3?

18. 37 taken from what number will leave 52?

19. 45 taken from what number will leave 34?

20. Two brothers brought home 95 trout and 17 perch; and of these the elder caught 62 fish: how many did the younger catch?

21. A lad having a dollar to spend on a holiday, paid 30 cents for a ride and 45 cents for a knife: how much did he have left?

22. The greater of two numbers is 57 and the difference is 45: what is the less?

13. Ex. 23. A lady having $40, paid $10 for a handkerchief and $18 for a hat: how much had she left?

24. A man's age was 29 yrs. 7 yrs. ago; his wife was 27 three yrs. ago: what is the difference of age now?

25. If a man spend 25 cents for cigars on a certain day, and 33 cents the next day, how much can he spend the third day to make up a dollar?

26. If a person was born in 1821, how old would he be in 1876?

27. What number must be taken from 28 + 46 to leave 37 remainder?

28. What must be added to 25 minus 17 to make 45?

29. In 1840 Sarah was 7 yrs. old: how old was she in 1873?

30. A man was 21 in 1850: how old is he now?

31. How old is a person who was born in 1803?

32. A and B entered into speculation; A furnished $120, and B $40 less than A: how much did both put in?

33. The greater of two numbers is 67, and the less is 45: what is the difference between them?

34. The difference between two numbers is 9, and the less number is 29: what is the greater number?

35. Two men made an adventure; one put in $175, the other $50 less: how much did they both furnish?

36. A frog at the bottom of a well 40 feet deep, jumped up 15 feet each day and fell back 5 feet the following night: how many days did it take him to reach the top?

CHAPTER III.

MULTIPLICATION.

14. Ex. 1. What will 7 hats cost, at $6 apiece?

Analysis.—Since 1 hat costs $6, 7 hats will cost 7 times $6, which are $42. Therefore, 7 hats will cost $42.

Or, thus: 7 hats will cost 7 times as much as 1 hat, and 7 times $6 are $42. *Ans.* $42.

NOTE.—Every example should be analyzed in a *clear*, *concise* and *logical* manner, and be recited with *distinctness* and *promptitude*.

2. How many trees are in 8 rows, each row containing 4 trees?

3. If Hellene writes 6 lines a day, how many lines will she write in 8 days?

4. If stage fare is 7 cents a mile, what will it cost to ride 10 miles?

5. What cost 6 combs, at 12 cents apiece?

6. Florence received 6 bouquets, each containing 11 flowers: how many flowers did she receive?

7. What will 8 boxes of raisins cost, at 7 shillings a box?

8. What is the weight of 6 turkeys, at 9 pounds apiece?

9. If George can skate 8 miles an hour, how far can he skate in 10 hours?

10. During the holidays Walter traveled 12 miles, and Henry 8 times as far: how far did Henry travel?

11. What cost 8 quarts of cherries, at 9 cents a quart?

12. At 7 cents each, what will 8 bananas cost?

13. A chessboard has 8 rows of squares, and 8 squares in a row: how many squares has it?

14. At $9 a dozen, what will 7 dozen caps cost?

15. What will a turkey weighing 9 pounds come to, at 12 cents a pound?

16. A coach goes 7 miles an hour: how far will it go in 11 hours?

17. In 1 pk. there are 8 qts.: how many qts. in 9 pecks?

18. At $11 a box, what will 9 boxes of oranges cost?

19. If you earn $9 a week, how much will you earn in 12 weeks?

20. At $12 a hundred, what cost 9 hundred melons?

21. Erastus sold 11 doves at 10 cents apiece: how much did he receive for them?

22. What will 12 dozen eggs cost, at 11 cents a dozen?

23. What is the process called by which the preceding examples are solved?

24. How many numbers are given in Multiplication, and what called?

Ans. Two; the ***Multiplicand*** and ***Multiplier.***

25. What else are they sometimes called?

Ans. ***Factors.***

26. What name or kind is the product?

Ans. It is the ***same name or kind*** as the multiplicand.

27. What must the multiplier be considered?

Ans. An ***Abstract Number.***

NOTE.—The product is the *same* in whatever order the numbers are multiplied. Thus: 3 times 5 = 5 times 3.

The *true multiplicand*, however, is that *factor* which, taken the given number of times, will produce the *required product.*

28. How is Multiplication denoted?

Ans. By ***an oblique cross*** called the ***Sign of Multiplication;*** as, ×.

Thus, the expression 5 × 3, shows that 5 and 3 are to be multiplied together. It is read "5 times 3," or "5 multiplied by 3."

29. $7 \times 6 =$ how many?
30. $5 \times 9 =$ how many?
31. $7 \times 8 =$ how many?
32. $11 \times 6 =$ how many?
33. $9 \times 6 =$ how many?
34. $7 \times 9 =$ how many?
35. $9 \times 8 =$ how many?
36. $12 \times 9 =$ how many?

15. Ex. 1. If 7 men can dig a trench in 8 days, how long will it take 1 man to dig it?

Analysis.—It will take 1 man 7 times as long as it will 7 men, and 7 times 8 days are 56 days. *Ans.* 56 days.

2. If a ton of hay will last 8 horses 6 days, how long will it last 1 horse?

3. If a barrel of bread will last 9 men 8 days, how long will it last 1 man?

4. If 9 men can build a barn in 12 days, how long will it take 1 man to build it?

5. If 1 man can lay 8 hundred bricks in a day, how many bricks will 9 men lay in the same time?

6. What will 12 readers cost, at 7 shillings apiece?

7. If the interest of $1 is 7 cents for a year, what will be the interest of $9 for the same time.

8. If the interest of 1 hundred dollars is $6 per annum, what must be the interest of 12 hundred dollars for the same time?

9. If 9 dollars earn 63 cents interest in 1 year, how long will it take 1 dollar to earn it?

10. If 12 men earn $96 in 4 days, how long will it take 1 man to earn it?

11. What two digits multiplied together produce 56? 63? 48? 45? 32? 27? 42? 54? 72?

12. What digit multiplied into itself makes 36? 25? 16? 49? 64? 81?

13. What number multiplied into itself produces 121? 100? 144? 400? 900?

14. A and B start from the same place and travel in opposite directions; A at the rate of 4, and B 5 miles an hour: how far apart will they be in 7 hours?

15. Two men start at the same place and travel in the same direction; one at the rate of 5, and the other 7 miles an hour: how far apart will they be in 10 hours?

16. In 1 pint there are 4 gills: how many gills are there in 11 pints?

17. In 1 peck there are 8 quarts: how many quarts are there in 12 pecks?

18. In 1 pound there are 16 ounces: how many ounces are there in 5 pounds?

17. How many days in 9 weeks?

16. Ex. 1. What will 4 muffs cost, at $60 apiece?

Analysis.—At $60 apiece, 4 muffs will cost 4 times $60. Now 60 is the same as 6 tens, and 4 times 6 tens are 24 tens or 240. Therefore, 4 muffs will cost $240, *Ans.*

2. At $50 a share, what will 5 shares of bank stock come to?

3. What will 7 tons of iron cost, at $60 per ton?

4. If a railroad car goes at the rate of 30 miles an hour, how far will it go in 9 hours?

5. How many are 4 times 40? 5 times 30? 3 times 60? 6 times 90? 7 times 80? 8 times 60?

6. How many are 9 times 80? 10 times 90? 8 times 50? 11 times 60? 12 times 80? 11 times 80?

7. A farmer bought 4 cows at $36 apiece: what did he pay for them all?

Analysis.—36 is the same as 3 tens and 6 units; 4 times 3 tens are 12 tens or 120, and 4 times 6 units are 24 units, which added to 120 make 144. Therefore, he paid $144 for his cows.

Or, thus: 4 times $30 are $120, and 4 times $6 are $24, which added to $120 make $144, *Ans.*

NOTE.—When the multiplicand exceeds 12, it is advisable in mental operations to begin with the highest order, as in addition. (Art. 4, Note.)

8. At 30 cents a bushel, what will 6 bushels of apples come to?

9. What cost 7 yards of muslin, at 18 cents a yard?

10. If a balloon goes at the rate of 25 miles an hour, how far will it go in 5 hours?

11. In an orchard there are 8 rows of trees, each row having 24 trees: how many trees are in the orchard?

12. At $37 per acre, what is the cost of 7 acres of land?

13. What cost 8 tons of hay, at $35 a ton?

14. A farmer wintered 27 cows, and 7 times as many sheep as cows: how many sheep did he winter?

15. At $75 apiece, what is the cost of 8 horses?

16. A certain garrison had provision to last a company of 60 men 9 days: how long will it last 1 man?

17. If a man can gather 57 bushels of apples in a day, how many bushels can he gather in 6 days?

18. What will 81 oranges cost, at 5 cents apiece?

19. If 9 men can do a piece of work in 40 days, how long will it take 1 man to do it?

20. At $85 a set, what will 10 sets of harness cost?

21. In 1 hour there are 60 minutes: how many minutes in 12 hours?

22. At $95 apiece, what will be the cost of 8 wagons?

23. Sold 11 pieces of muslin, each containing 42 yards: how many yards were sold?

24. If a boy earns 87 cents a day, how much will he earn in 6 days?

25. A boat's crew of 12 whalemen being cast upon a desert island, had bread sufficient to last them 45 days: how long would it last 1 man?

26. At $97 a share, what will 5 bank shares cost me?

27. What will 10 yearlings cost, at $17 apiece?

28. If there are 700 men in 1 battalion, how many men are there in 10 battalions?

29. In 1 hogshead there are 63 gallons: how many gallons are there in 100 hogsheads?

30. A grocer bought 40 barrels of apples at $5 a barrel, and sold them at a loss of $20: what did he get for them?

17. Finding the Product of the Sum of two or more Numbers which have the same Multiplier.

Ex. 1. If apples are 2 cents apiece, plums 3 cents, and pears 4 cents, how much must you pay for 7 of each?

Analysis.—2 cents + 3 cents + 4 cents are 9 cents; hence one of each would cost 9 cents. Now if one of each cost 9 cents, 7 of each would cost 7 times 9 cents, which are 63 cents, *Ans.*

2. If Gertrude gains 5 credits a day and Agnes 7 credits, how many credits will both gain in 6 days?

3. Two men starting at the same place and time, travel in opposite directions; one goes 3 miles an hour, the other 4 miles: how far apart will they be in 12 hours?

4. A merchant has cloths worth $2, $3, and $4 a yard respectively: what must I pay for 12 yards of each?

5. A farmer sold 5 loads of hay to one man, 3 loads to another, and 2 loads to another: their average weight was 14 hundred pounds: how much did they all weigh?

6. Two men being 100 miles apart, travel towards each other; one at the rate of 4 miles an hour, the other at the rate of 5 miles: how far apart will they be at the end of 10 hours?

7. A housekeeper bought 3 dozen eggs at one place, 2 dozen at another, and 6 dozen at another, and paid 9 cents a dozen: what did they all cost her?

8. How many are 6 times 5 + 3 + 4?

9. How many are 9 times 12 plus 7?

10. How many are 7 times 8 plus 20?

11. How many are 11 times 12 plus 30?

12. A farmer sold 8 barrels of cranberries at $9 a barrel, and charged $2 a barrel for delivering them: what was the amount of his bill?

13. A merchant imported 7 pieces of silk at $50 apiece, and the duty was $25 on each piece: what did the silk cost him?

18. Drill Exercises in the preceding Rules.

Ex. 1. To 13 add 7, subtract 5, multiply by 2, subtract 15, subtract 10, multiply by 3, subtract 8: result?

2. From 15 subtract 9, multiply by 3, subtract 8, add 5, multiply by 2, subtract 20, add 8, multiply by 2, subtract 9, add 8: result?

3. Multiply 12 by 5, subtract 40, add 5, multiply by 2, subtract 25, add 5, multiply by 3, add 9: result?

4. Add 7 to 9, subtract 6, multiply by 4, subtract 20, add 7, subtract 5, multiply by 2, subtract 8, add 9: result?

5. Subtract 8 from 17, multiply by 5, subtract 15, multiply by 20, subtract 30, add 9, subtract 7: result?

6. To the product of 8 into 8 add 6, subtract 30, add 2, subtract 12, multiply by 3, subtract 4, add 16: result?

7. From 16 subtract 9, multiply by 3, subtract 7, add 4, multiply by 6, subtract 7, add 9, subtract 6: result?

8. Add 6 to 18, subtract 9, multiply by 4, subtract 25, multiply by 2, subtract 40, multiply by 8: result?

9. From 19 subtract 8, multiply by 6, subtract 11, add 7, subtract 20, add 8, multiply by 3, add 7: result?

10. Multiply 7 by 6, subtract 12, add 4, subtract 14, multiply by 6, subtract 20, multiply by 3, subtract 12, add 37: result?

11. To 19 add 11, subtract 15, multiply by 4, subtract 12, multiply by 2, add 9, subtract 5, multiply by 6: result?

12. Subtract 9 from 21, add 8, subtract 6, add 11, multiply by 4, subtract 7, add 9, subtract 4: result?

13. To the product of 9 into 6, add 6, subtract 12, subtract 18, multiply by 2, subtract 20, add 5, multiply by 2, add 10, multiply by 3, add 15, subtract 7, add 8: result?

14. From 26 subtract 8, multiply by 2, add 4, multiply by 4, subtract 40, subtract 11, add 6, multiply by 2, subtract 20, multiply by 3, add 8, subtract 7, add 9, subtract 6, add 8: result?

QUESTIONS FOR REVIEW.

19. Ex. 1. A grocer bought 50 barrels of apples, at $3 a barrel, and sold them, at a profit of $2 a barrel: what did he get for his apples?

2. Horace bought 20 rabbits, at 12 cents apiece, and sold them so as to lose 3 cents on each: how much did he get for them?

3. A drover bought 12 lambs, at 9 shillings a head, and afterwards sold them, at 12 shillings a head: how much did he make by his bargain?

4. A man bought 11 cows, at $20 apiece, and paid 10 tons of hay, at $15 a ton: how much did he owe?

5. What is the difference between 11×7, and 9×8?

6. What is the difference between 14×10, and 13×11?

7. A carpenter earns 9 shillings a day, and a mason 11 shillings: what is the difference in their wages in 12 days?

8. A farmer having 75 turkeys, sold 50 of them, at 6 shillings apiece, and the rest, at 4 shillings apiece: what did they come to?

9. A man owns 3 orchards; in each orchard there are 8 rows of apple-trees, and 20 trees in each row: how many apple-trees has he?

10. In a certain school there are 3 departments, in each department there are 7 classes, and in each class there are 20 pupils: how many pupils are there in the school?

11. A man sent 10 loads of wheat to market; every load contained 12 bags, and every bag 3 bushels: how many bushels did he send?

12. George has 17 cents, and Henry has 3 times as many as George lacking 5: how many cents has Henry? How many have both?

13. At a military parade there were 5 regiments, in each regiment 8 companies, and in each company 100 soldiers: how many soldiers were on parade?

CHAPTER IV.

DIVISION.

20. Finding how many times One number is contained in Another.

Ex. 1. How many times is 8 contained in 40?

Analysis.—Since 5 times 8 are 40, it follows that 8 is contained in 40, 5 times, *Ans.*

2. How many times is 9 contained in 45? 6 in 54?

3. How many times is 8 contained in 56? 7 in 63?

4. How many melons, at 7 cents apiece, can be bought for 28 cents?

Analysis.—Since 7 cents will buy 1 melon, 28 cents will buy as many melons as 7 cents are contained times in 28 cents; and 7 cents are contained in 28 cents 4 times. *Ans.* 4 melons.

Or, thus: at 7 cents each, 28 cents will buy as many melons as there are 7s in 28, or 4 melons, *Ans.*

5. If a stage coach goes at the rate of 8 miles an hour, how long will it be in going 48 miles?

6. When the car fare is 6 cents a ride, how many rides can you take for 54 cents?

7. How many barrels of pears, at $9 a barrel, can be bought for $108?

8. If a farmer packs 12 pounds of butter in a jar, how many jars will he require to pack 96 pounds?

9. At $8 a ton, how much coal can be bought for $72?

10. A man has $60 which he wishes to lay out in silver watches, at $5 apiece: how many watches can he buy?

11. How many casks of nails, at $7 a cask, can be bought for $63?

12. If a man can earn $9 a week, how long will it take him to earn $54?

13. How many bushels of filberts, at $8 a bushel, can you buy for $56?

14. If you drive at the rate of 9 miles an hour, how long will it take you to go 72 miles?

15. At $12 a hundred, how much flax can be bought for $132?

NOTE.—The object of each of the preceding examples is, to *find how many times* one number is contained in another of the *same name or kind.* Hence, the result being *times,* is an *abstract* number.

21. Dividing Numbers into Equal Parts.

1. A man divided $72 equally among his 6 sons: how many dollars did each receive?

Analysis.—Since $72 were divided equally among 6 sons, each son must have received $1 as often as 6 is contained times in 72, which is 12 times. *Ans.* $12.

2. A teacher having 66 pens, distributed them equally in a class of 11 pupils: how many did each get?

3. If 96 pounds of sugar are divided equally among 12 persons, how many pounds will each receive?

4. If 88 candles are divided equally among 8 persons, how many candles will each have?

5. If 120 pounds of flour are divided equally among 12 families, how much flour will each receive?

6. A teacher having 108 pupils, divided them into 9 equal classes: how many were in each class?

7. A sleighing party of 11 persons spent $132: how much was that apiece?

8. A company of 10 persons found a purse containing $100, which they shared equally: how much did each receive?

9. A lad having $96, wishes to divide it equally among 8 benevolent societies: how much can he give to each?

10. If you pay 84 cents for a horse and chaise, to go 7 miles, how much is that a mile?

11. A grocer having 56 dozen eggs, packed them in 8 boxes: how many dozen did he put in each?

12. If $132 are divided into 12 equal parts, how many dollars will there be in each part?

NOTES.—1. The object of each example in the last article is to *divide* a number into *equal parts*. The given numbers it will be seen are of *different* names, and the answer is the *same* name as the *number divided*.

2. When a number or thing is divided into *two* equal parts, the parts are called *halves*; when into *three* equal parts, the parts are called *thirds*; when into *four*, they are called *fourths*, etc.

The *name* of the parts is determined by their *number*.

13. What is the process called by which the examples in the last two Articles are solved?

14. How many numbers are given in Division?

Ans. Two; the ***Divisor*** and ***Dividend***.

15. What is the answer called?

Ans. The ***Quotient***.

16. What is the number which is sometimes left after division, called?

Ans. The ***Remainder***.

17. When we say 3 is contained in 17, 5 times and 2 over, what is the 3 called? The 17? The 5? The 2?

18. Describe the sign of Division.

Ans. It is a ***short horizontal line*** between the points of a colon; as, $\div$.

Thus, $6 \div 3$ shows that the number before the sign is to be divided by the one after it; and is read "six divided by three."

19. How else is division denoted?

Ans. By writing the ***divisor under the dividend*** with a short line between them; as, $\frac{6}{3}$, which is read, "6 divided 3," or "6 thirds."

20. $27 \div 9 =$ how many?
21. $35 \div 7 =$ how many?
22. $56 \div 8 =$ how many?
23. $54 \div 6 =$ how many?
24. $63 \div 7 =$ how many?
25. $72 \div 9 =$ how ma

22. Ex. 1. If 63 pears are divided equally among 7 girls, how many will each receive?

Analysis.—One is 1 seventh of 7; therefore 1 girl will receive 1 seventh of 63 pears, which is 9 pears. *Ans.* 9 pears.

2. A man divided 42 peaches equally among 6 boys, how many did each receive?

3. A farm of 63 acres is divided into 9 fields: how many acres in each field?

4. I have 72 peach trees which I wish to set in 8 equal rows: how many must I put in a row?

5. If 12 settees will seat 108 persons, how many will 1 settee accommodate?

6. If a man travels 80 miles in 10 hours, how many miles will he travel in 1 hour?

7. In 3 times 14, how many times 6?

8. In 6 times 8, how many times 12?

9. In 3 times 15, how many times 9?

10. In 12 times 6, how many times 8?

11. At 10 cents apiece, how many slates can you buy for $1?

12. At $11 each, how many watches can be bought for $121?

13. At 12 cents apiece, how many bouquets can you buy for 96 cents?

14. A company of 9 boys found a purse containing $72, and shared it equally: what was each one's share?

15. 24 is double what number? Triple? Quadruple?

16. By what number must 80 be divided, that the quotient may be 8?

17. By what number must 100 be divided that the quotient may be 10? 20? 25? 50?

18. A man having 110 acres of land, divided it into 6 equal lots: how many acres in each lot?

19. A farmer had 132 bushels of wheat, which he put into 12 equal bins: how many bushels in each bin?

23. Ex. 1. A market-man has 67 eggs which he wishes to put in baskets containing 12 each: how many baskets will he require, and how many eggs will he have over?

Analysis.—Since he puts 12 eggs in a basket, he will require as many baskets as there are 12s in 67, which are 5. But 5 times 12 are 60, and 60 from 67 leaves 7. Therefore he will require 5 baskets, and have 7 eggs over.

NOTE.—As the learner is not supposed to be acquainted with fractions, the *remainder*, if any, may be mentioned as such.

2. A farmer had $38 which he wished to lay out in sheep, at $5 each: how many could he buy, and how much money would he have left?

3. George has 75 cents which he wishes to exchange for dimes; how many dimes should he receive, and how many cents over?

4. How many times 6 in 45, and what remainder?

5. How many times 7 in 51? In 56? In 61? In 70?

6. How many times 8 in 39? In 45? In 56? In 67? In 72?

7. In 38, how many times 9? In 45? In 50? In 63? In 74?

8. James having 68 cents, bought 7 balls, and had 5 cents left: what were the balls apiece?

9. How many 3-cent postage stamps can you obtain for 40 cents?

10. How many 5-cent pieces can you obtain for 39 cents?

11. How many 8-dollar revenue stamps can be obtained for $67?

12. How many 8-cent loaves of bread can be made out of 75 cents worth of flour?

13. How many times 7 in 8 times 8, and how many over?

14. How many times 9 in 7 times 8?

15. In 7 times 9, how many times 6?

16. In 8 times 11, how many times 9?

17. In 9 times 12, how many times 11?

18. In 75, how many times 12? How many times 10? How many times 9? How many times 8?

19. How many acres of land, at $11 an acre, can be bought with $112?

20. At $12 apiece, how many engravings can be bought for $150?

21. How many sixpences are in 79 pence?

22. A man sent his servant with 25 cents to buy as many 3-cent postage stamps as he could, and take the rest in 2-cent stamps so as to leave no remainder: how many of each did he get?

23. If Charles has 65 cents and his mother gives him 7 more, how many bananas can he buy at 6 cents apiece?

Analysis.—65 cts. + 7 cts. are 72 cts. Now, if 6 cts. will buy 1 banana, 72 cts. will buy as many as there are 6s in 72, which are 12. *Ans.* 12 bananas.

24. How many times 6 in 24 plus 12?
25. How many times 7 in 42 plus 8?
26. How many times 8 in 58 plus 3?
27. How many times 9 in 64 plus 5?
28. How many times 12 in 83 plus 9?
29. In 59 minus 3, how many times 7?
30. In 67 minus 4, how many times 9?
31. In 78 minus 6, how many times 8?
32. In 93 — 12, how many times 9?
33. In 109 — 7, how many times 10?
34. In 108 — 9, how many times 12?
35. In 5 times 9 + 15, how many times 8 + 4?
36. In 6 times 9 + 12, how many times 6 + 2?
37. In 7 times 12 + 16, how many times 5 + 6?
38. In 9 times 8 — 12, how many times 20 — 5?
39. In 8 times 12 — 8, how many times 17 — 6?
40. How many times 16 minus 5 in 8 times 7 plus 12?
41. How many times 20 minus 9 in 120 plus 12?

24. Ex. 1. At $30 apiece, how many cows can a man buy for $150?

Analysis.—30 is 3 tens, and 150 is 15 tens; now 3 tens are contained in 15 tens, 5 times. *Ans.* 5 cows.

2. At $40 an acre, how many acres of land can you buy for $120?

3. In 80, how many times 20? How many times 40?

4. In 160, how many times 20? How many times 40? How many times 80?

5. How many colts at $50 each, can a man buy for $300?

6. In 120, how many times 20? How many times 30? How many times 40? How many times 60?

7. In 420, how many times 60? How many times 70?

8. A farmer sowed 40 acres of wheat, which yielded 1200 bushels: how much was that per acre?

9. In 97, how many times 30, and how many over? In 125? In 156? In 188? In 215?

10. A drover bought 200 sheep for $1000: what was that a head?

11. A miller sold 300 barrels of flour for $2100: how much did he receive per barrel?

12. A farm of 400 acres was sold for $16000: what was that per acre?

13. How many horses at $80 apiece, can you buy for $640?

14. How many barrels of beef in 6800 pounds, allowing 200 pounds to the barrel?

15. How many regiments of 4000 men each can be formed from 24000 soldiers?

16. How many farms of 80 acres each can be formed from 1600 acres of land?

17. At $70 apiece, how many horses can a jockey buy for $2100?

18. In a cu. ft. of water there are 1000 ounces: how many cu. ft. are in 12000 ounces?

DRILL FOR RAPID COMBINATIONS.

NOTE.—Dictation Exercises like the following, are among the best means for securing *accuracy* and *rapidity* in the simple combinations of numbers. If conducted briskly, they also impart more *power* of *abstraction* and *mental concentration* in fifteen minutes, than is gained in as many hours from reciting ordinary examples in a slipshod manner.

25. Ex. 1. To 5 add 7, multiply by 3, subtract 6, divide by 5, multiply by 8, divide by 4, add 6: what is the result?

2. From 15 take 8, multiply by 6, divide by 7, add 10, divide by 8, add 20, take 4, divide by 3, multiply by 9: what is the result?

3. Multiply 7 by 8, take 2, divide by 6, add 7, divide by 4, add 26, divide by 5, multiply by 7, add 6, divide by 8, multiply by 9: result?

4. Divide 45 by 5, multiply by 3, add 8, divide by 7, add 31, take 4, divide by 8, multiply by 9, add 6, divide by 7, add 15, take 9, multiply by 7: result?

5. Add 9 to 19, divide by 7, multiply by 8, take 7, divide by 5, multiply by 12, take 4, divide by 8, add 17: result?

6. Subtract 7 from 25, divide by 6, multiply by 9, add 8, divide by 7, multiply by 20, take 4, divide by 12, multiply by 6: result?

7. To the product of 7 into 5, add 9, divide by 11, multiply by 12, take 3, divide by 9, multiply by 10, add 6, divide by 7, add 8, divide by 2, add 19, divide by 9, multiply by 15: result?

8. To the quotient of 63 divided by 7, add 6, multiply by 4, divide by 12, add 30, divide by 7, add 16, divide by 7, multiply by 11, add 9, divide by 7, add 15, take 7, divide by 7, add 35: result?

9. To the difference between 7 and 15, add 10, divide by 6, multiply by 11, add 9, divide by 7, multiply by 9, take 6, divide by 8, multiply by 7, add 8: result?

10. To 23 add 9, divide by 8, add 35, divide by 3, add 8, divide by 7, multiply by 20, divide by 10, add 27, take 9, divide by 8, multiply by 15, add 9: result?

11. From 41 take 5, divide by 9, multiply by 11, add 12, divide by 8, multiply by 7, take 5, divide by 4, multiply by 7, take 5, divide by 9, add 34, divide by 7: result?

12. Add 35 to 9, divide by 11, multiply by 25, take 16, divide by 12, add 43, divide by 5, add 53, take 13, divide by 5: result?

13. Multiply 7 by 8, add 10, divide by 11, add 21, divide by 9, multiply by 12, add 12, divide by 8, multiply by 11, divide by 6: result?

14. Divide 72 by 9, multiply by 7, take 8, divide by 6, multiply by 12, add 12, divide by 9, add 52, divide by 8, multiply by 12, divide by 8: result?

15. To 61 add 11, divide by 6, take 11, add 55, divide by 7, multiply by 6, take 18, divide by 6, multiply by 30, take 6, divide by 12: result?

16. From 85 take 15, divide by 7, multiply by 8, add 16, divide by 8, add 30, divide by 7, multiply by 20, add 12, divide by 11: result?

17. Multiply 30 by 4, divide by 12, add 25, divide by 7, multiply by 11, add 9, divide by 8, multiply by 5, take 7, add 23, divide by 8: result?

18. Add 17 to 20, take 9, divide by 7, multiply by 25, take 4, divide by 8, add 8, multiply by 4, take 20, add 30, divide by 10, multiply by 8, add 12, divide by 12, add 7: result?

19. Divide 56 by 8, multiply by 9, divide by 7, multiply by 8, add 5, divide by 11, multiply by 6, add 21, divide by 9, add 20, divide by 3, multiply by 5, add 15, divide by 10, multiply by 8, divide by 12: result?

QUESTIONS FOR REVIEW.

26. Ex. 1. A father bought 16 oranges at one place, 9 at another, and 15 at another, and divided them equally among his 5 children: how many did each receive?

2. A grocer bought 12 lbs. of butter of one customer, 20 lbs. of another, 8 lbs. of another, and 32 lbs. of another; he packed it in jars holding 9 lbs. each: how many jars did he require?

3. A farmer gathered 14 bushels of apples from one tree, 12 from another, 10 from another, and 4 from another: how many barrels of cider can he make, allowing 8 bushels of apples to a barrel?

4. A tailor bought 3 pieces of cloth containing 17 yds., 20 yds., and 13 yds., and made them into cloaks containing 5 yards each: how many cloaks did he make?

5. A man having 7 times 8 apples, plus 4, divided them equally among 12 boys: how many did each receive?

6. What number is that, to which if 4 times 5 be added, the sum will be 108?

7. What number is that, to which if 6 times 7 be added, the sum will be 145?

8. What number is that, which if divided by 3 times 4, the quotient will be 8?

9. What number is that, which if divided by 4 times 5, the quotient will be 6?

10. A lad having 54 peaches, lost 12 of them on his way to school, and divided the rest equally among 7 of his companions: how many did each receive?

11. A man having a hundred dollar bill, paid $16 for a cow, and laid out the rest in hay at $12 per ton; how much hay did he buy?

12. A man bought 12 boxes of raspberries, containing 4 quarts apiece, which he distributed equally among 8 poor families: how many quarts did each receive?

13. How many times are 3 times 12 contained in 9 times 8?

14. How many feet in 4 times 15 inches?

15. How many yards in 8 times 10 feet?

16. How many times can 4 be subtracted from 6 times 10? How many times can 5? 12? 10? 6? 15? 20?

17. From 9 times 8 how many times can 6 be subtracted?

18. What number is that, the factors of which are 7, 9, and 20?

19. What number is that, the factors of which are 5, 7, and 8?

20. What number besides 13 will exactly divide 39?

21. What number besides 17 will exactly divide 85?

22. When wood is $4 a cord and coal is $9 a ton, how much wood is equal in value to 8 tons of coal?

23. If eggs are worth 9 cents a dozen, and butter 12 cents a pound, how many eggs are worth 6 lbs. of butter?

24. A man being on a journey, found he could reach home in 9 days by traveling 20 miles a day; but becoming lame, he traveled only 12 miles a day: in how many days did he reach home?

25. A lady had just money enough to buy 16 yds. of silk, at 9 shillings per yard; but she found the silk was 12 shillings a yard: how much money had she? How many yards could she buy?

26. Two men being 132 miles apart, start to meet each other, one traveling 6 miles an hour, and the other 5 miles an hour: how many hours before they will meet? How many miles will each have traveled? And how much farther will one have traveled than the other?

27. What number of apples must be divided among 12 boys, that they may receive 25 apiece?

28. What number of dollars must be given to 9 beggars, that each may receive $38 less $7?

29. John has 12 marbles and William has 9 times as many as John, minus 11: how many marbles has William? How many have both?

30. When peaches are selling at the rate of 5 for 8 cents, how many will 56 cents buy?

31. What cost 60 apples, at the rate of 10 for 7 cents?

32. George bought 12 oranges, at 4 cents apiece, and after eating 3 of them, sold the rest, at 6 cents apiece: did he make or lose by his bargain, and how much?

33. Henry sold 4 doves at 20 cents apiece; he took 3 penknives at 16 cents apiece, and the rest in money: how much money did he receive?

34. If you spend 48 cents for eggs, at the rate of 8 for 6 cents, and sell them at the rate of 8 for 7 cents, how much will you make by the operation?

35. A drover bought 36 lambs, at the rate of 6 for $7, and sold them, at the rate of 12 for $10: how much did he lose by the operation?

36. Two men, A and B, are 36 miles apart, and are traveling in the same direction; A goes 3 miles an hour, and B 5 miles: how long will it take B to overtake A?

37. How many tons of coal, worth $6 a ton, will it take to pay for 12 barrels of flour worth $8 a barrel?

38. A and B are 55 miles apart, and start to meet each other; A travels 7 miles an hour, which is 3 miles more than B travels: in how many hours will they meet: and how much farther will A have traveled than B?

39. A man bought 6 hats at $4 apiece, and 5 caps at $2, and paid for them in flour at $8 a barrel: how many barrels of flour and how much money did it take to pay the bill?

40. A father and son started on a journey of 168 miles, which the father performed in 8 days: how long would it take the son, if he traveled 9 miles a day less than his father?

CHAPTER V.

FACTORING.

27. Finding the Factors of a Number.

1. What is meant by Factoring a number?

Ans. It is finding two or more numbers which, multiplied together, produce the given number.

2. What are the factors of 14?

Analysis.—7 multiplied by 2 produces 14. Hence, 2 and 7 are the factors of 14.

3. What are the factors of 6? Of 9? 10? 15?

4. What are the factors of 21? Of 22? 26? 25? 33?

5. What are the factors of 35? 49? 55? 77? 121?

NOTE.—Some numbers may be resolved into *more* than *two* factors; and also into *different sets* of factors. Thus, 12 may be resolved into the factors 3, 2, and 2; 4 and 3; and 6 and 2.

6. What are the different factors of 18?

Analysis.—18 = 9 into 2; or 6 into 3; or 3 into 3 into 2. Therefore, the different factors of 18 are 9 and 2; 6 and 3; and 3, 3, and 2.

7. Resolve 16 into two factors. Into three factors. Into four factors.

8. What three factors multiplied together produce 28?

9. Resolve 20 into two factors. Into three.

10. Resolve 30 into 3 factors.

11. Resolve 36 into the greatest number of factors possible.

12. Name three factors of 32? Of 40? 54?

13. Name all the factors of 42? Of 60? 72?

28. Ex. 1. What is an exact divisor?

Ans. An ***Exact Divisor*** is a number that will divide another number without a *remainder.*

NOTES.—1. An *exact divisor* of a number is always a *factor* of that number; and, conversely, a *factor* of a number is always an *exact divisor* of it.

2. When it is required to find *all the factors or exact divisors* of a number, it is advisable to begin with the *smallest*, and proceed in regular order.

2. What numbers will divide 28 without a remainder?

3. What are the exact divisors of 30? 32? 36? 40?

4. Name the exact divisors of 24? 48? 50? 55? 60?

5. What numbers under 42 are divisible by 3?

6. What numbers under 65 are exactly divisible by 4? By 5? By 6?

7. What numbers under 85 are exactly divisible by 7? By 8? By 9?

8. What is a common divisor?

Ans. A ***Common Divisor*** is any number that will divide *two* or *more* numbers without a *remainder.*

9. What is a common divisor of 15 and 21?

Analysis.—By inspection we see that 15 and 21 are each exactly divisible by 3; therefore, 3 is the common divisor required.

NOTE.—The exercise of finding the factors, common divisors, common multiples, etc., of numbers mentally, is *invaluable* in developing the *power* of inspection; an attainment which can only be acquired by practice.

10. Name a common divisor of 24 and 30. Of 27 and 35. Of 42 and 63. Of 54 and 63.

11. Name all the common divisors of 12, 18, and 30.

12. Name all the common divisors of 24, 40, and 60.

13. Name all the common divisors of 36, 48, 54, and 72.

14. What is the greatest common divisor?

Ans. It is the ***greatest number*** that will divide two or more numbers without a remainder.

15. What is the greatest common divisor of 18, 24, and 30?

Analysis.—We see by inspection that 2, 3, and 6 are common divisors of 18, 24, and 30, and that no other number will divide them all; therefore the greatest common divisor is 6.

16. What is the greatest common divisor of 16, 24, and 36?

17. What is the greatest com. div. of 9, 27, and 33?

18. What is the greatest com. div. of 16, 32, and 48?

19. What is the greatest com. div. of 15, 35, and 56?

20. What is the greatest com. div. of 18, 42, and 63?

MULTIPLES.

29. Finding Multiples of Numbers.

1. What is a Multiple?

Ans. A ***Multiple*** is a number which can be divided by another number *without a remainder.*

2. What numbers less than 24 are multiples of 3?

Ans. 6, 9, 12, 15, 18, 21.

3. What numbers under 24 are multiples of 2?

4. What numbers under 48 are multiples of 4?

5. What numbers under 60 are multiples of 5?

6. What numbers under 72 are multiples of 6?

7. What is a common multiple?

Ans. A ***Common Multiple*** is a number which can be divided by *two* or *more* numbers without a remainder.

Thus, 15 is a common multiple of 3 and 5.

NOTE.—A common multiple of two or more numbers may be found by multiplying the numbers together.

8. Of what two numbers is 6 a multiple?

9. Of what two numbers is 15 a multiple?

10. Of what two numbers is 21 a multiple?

11. Of what two numbers is 35 a multiple? Is 55 a multiple? Is 77?

12. Name all the numbers of which 12 is a common multiple.

13. Name all the numbers of which 18 is a common multiple.

14. Of what numbers is 24 a common multiple?

15. Of what numbers is 30 a common multiple? Is 34? 36? 40?

16. Of what is 42 a common multiple? 45? 48? 60? 63? 75? 84? 108? 77? 132?

17. What is a common multiple of 3 and 11? Of 5 and 9? Of 7 and 6?

18. What is a common multiple of 7 and 8? Of 8 and 9? Of 7 and 12?

19. What is the *Least Common Multiple* of two or more numbers?

Ans. It is the ***Least Number*** that can be divided by each of them without a remainder.

Thus, 15 is the least common multiple of 3 and 5; 28 is the least common multiple of 4 and 7.

20. What is the least common multiple of 8 and 12?

21. What is the least common multiple of 9 and 6?

22. What is the least common multiple of 2, 4, and 6?

23. What is the least common multiple of 4, 12, and 16?

24. What number is the least common multiple of 6, 8 and 12?

25. What number is the least common multiple of 5, 12, and 15.

CHAPTER VI.

FRACTIONS.

30. Ex. 1. When a number or thing is divided into *two* equal parts, what is *one* of the parts called?

2. If divided into *three* equal parts, what? If into *four*, what? If into *five*? Into *ten*? A *hundred*?

3. From what do these parts take their name?

Ans. From ***the Number of parts*** into which the unit is divided.

4. How many *thirds* in a *unit* or *one*? How many fifths? Sevenths? Tenths? Thirtieths? Hundredths?

5. What is meant by one third? Two thirds?

6. What is meant by a fourth? Three fourths?

7. What are fourths sometimes called?

8. Which is the less, *thirds* or *fourths*? Why?

Ans. Because a ***fourth*** is one of the *four*, and a ***third*** is one of the *three* equal parts into which a unit is divided; and the *greater* the *number* of parts a thing is divided into, the *less* will be *each part*.

9. Which is greater, fifths or fourths? Sixths or sevenths? Elevenths or ninths?

10. When a unit is divided into two or more equal parts, as halves, thirds, etc., what are these expressions called?

Ans. ***Fractions.***

11. From what do fractions arise?

Ans. From ***Division;*** the *numerator* being the *dividend* and the *denominator* the *divisor*.

12. What is the *value* of a fraction?

Ans. The ***Quotient*** of the numerator divided by the denominator. Thus, the value of $\frac{6}{3}$ is $6 \div 3 = 2$.

13. How are fractions commonly expressed?

Ans. By ***Figures*** written *above* and *below* a short line.

14. What are the figures below the line called, and why?

Ans. The ***Denominator,*** because the name shows into *how many equal* parts the unit is divided.

15. What are the figures above the line called, and why?

Ans. The ***Numerator,*** because they *number* the parts, showing *how many* are expressed by the fraction.

16. What are the numerator and denominator together called?

Ans. The ***Terms*** of the fraction.

17. How express five-eighths by figures? Eight-tenths?

18. How express two-ninths? Seven-twelfths? Seventeen-nineteenths? Twenty-three hundredths?

19. Explain the fraction $\frac{3}{4}$?

Analysis.—$\frac{3}{4}$ denotes $\frac{3}{4}$ of 1, or $\frac{1}{4}$ of 3. For if 3 equal things, as 3 apples, are each divided into 4 parts, 3 of these parts will be 3 fourths of 1 apple, or 1 fourth of 3 apples. The denominator 4, names the parts, and the numerator 3, shows that 3 parts are taken.

Explain the following fractions:

$\frac{5}{8}$; $\frac{4}{7}$; $\frac{10}{9}$; $\frac{8}{5}$; $\frac{24}{30}$; $\frac{35}{63}$; $\frac{75}{25}$; $\frac{84}{43}$; $\frac{99}{100}$.

NOTE.—Care should be taken to see that the language employed in fractions is fully understood by the pupil. The operations will then be essentially as easy as those in whole numbers.

31. Finding a Fractional part of a Number.

1. What is 1 half of 2 dollars?

Analysis.—If 2 dollars are divided into two equal parts, one of those parts is 1 dollar. Therefore, 1 half of $2 is 1 dollar.

2. What is 1 half of 6? Of 10? Of 18? Of 22?

3. What is 1 fourth of 8? Of 12? Of 18? Of 24? Of 32? Of 44?

4. What is 1 fifth of 10? 1 sixth of 18? 1 seventh of 35? 1 eighth of 40? 1 ninth of 54?

5. What are 2 thirds of 3 dollars?

Analysis.—2 thirds of $3 are twice as much as 1 third. But 1 third of $3 is $1; therefore, 2 thirds of $3 are $2.

6. What are $\frac{3}{4}$ of 8?

Ans. $\frac{1}{4}$ of 8 is 2, and $\frac{3}{4}$ are 3 times 2 or 6.

Or, thus: $\frac{1}{4}$ of 1 is $\frac{1}{4}$, and $\frac{1}{4}$ of 8 is 8 times $\frac{1}{4}$ of 1 or $\frac{8}{4} = 2$, and $\frac{3}{4}$ of 8 are 3 times 2 or 6, *Ans.*

7. What are $\frac{2}{5}$ of 10? $\frac{3}{5}$ of 15? $\frac{4}{5}$ of 25?
8. What are $\frac{2}{6}$ of 12? $\frac{4}{6}$ of 18? $\frac{5}{6}$ of 42?
9. What are $\frac{3}{7}$ of 21? $\frac{4}{7}$ of 35? $\frac{6}{7}$ of 49?
10. What are $\frac{3}{8}$ of 24? $\frac{5}{8}$ of 45? $\frac{7}{10}$ of 60?
11. What are $\frac{5}{11}$ of 66? $\frac{8}{12}$ of 96? $\frac{2}{7}$ of 63?

32. General Principles of Fractions.

1. What is the value of the fraction $\frac{2}{4}$?

Ans. It is 2 divided by 4, or 1 half.

2. If the numerator 2 is *multiplied* by 2, what is the value of the fraction?

Ans. It is 4 divided by 4, or 1.

3. If the denominator 4 is *divided* by 2, what is the value?

Ans. It is 2 divided by 2, or 1.

33. What then is the effect of *multiplying* the numerator, or *dividing* the denominator?

Ans. It ***multiplies*** the fraction.

4. If the numerator of $\frac{2}{4}$ is *divided* by 2, what is its value?

Ans. It is 1 divided by 4, or 1 fourth.

5. If its denominator is *multiplied* by 2, what is the value?

Ans. It is 2 divided by 8, or $\frac{2}{8} = 1$ fourth.

34. What then is the effect of *dividing* the numerator, or *multiplying* the denominator?

Ans. It ***divides*** the fraction.

6. If both terms of $\frac{2}{4}$ are multiplied by 2, what is its value?

Ans. It is (2×2) divided by $(4 \times 2) = \frac{4}{8}$ or $\frac{1}{2}$.

7. If both terms are divided by 2, what is its value?

Ans. It is $(2 \div 2)$ divided by $(4 \div 2) = \frac{1}{2}$.

35. What then is the effect if the terms of a fraction are both *multiplied* or both *divided* by the same number?

Ans. Its ***value*** is not altered.

REDUCTION OF FRACTIONS.

36. Reducing Fractions to their Lowest terms.

1. When is a fraction reduced to its lowest terms?

Ans. When *no number greater* than 1 will divide each of them without a *remainder*.

2. What are the lowest terms to which $\frac{16}{24}$ can be reduced?

Analysis.—Since dividing both terms of a fraction does not change its value, it follows that dividing both terms of $\frac{16}{24}$ by 2 it becomes $\frac{8}{12}$. (Art. 35.)

Again, dividing both terms of $\frac{8}{12}$ by 2 it becomes $\frac{4}{6}$; finally, dividing both terms of $\frac{4}{6}$ by 2 it becomes $\frac{2}{3}$, which are the lowest terms in which $\frac{16}{24}$ can be expressed. (Prac. Arith., Art. 144.)

Or, we may divide both terms of $\frac{16}{24}$ by 8, their greatest common divisor, and the result is $\frac{2}{3}$, the same as before.

Reduce the following fractions to their lowest terms:

3. $\frac{10}{15}$.
4. $\frac{18}{24}$.
5. $\frac{2}{6}$.
6. $\frac{20}{12}$.
7. $\frac{21}{28}$.
8. $\frac{28}{42}$.
9. $\frac{35}{49}$.
10. $\frac{40}{60}$.
11. $\frac{17}{19}$.
12. $\frac{25}{100}$.
13. $\frac{50}{75}$.
14. $\frac{75}{100}$.

37. Reducing Fractions to any required Denominator.

1. Reduce $\frac{3}{4}$ to twelfths without altering its value.

Analysis.—In 1 there are 12 twelfths; hence, in $\frac{1}{4}$ of 1 there must be $\frac{1}{4}$ of 12 twelfths. Now $\frac{1}{4}$ of 12 twelfths is 3 twelfths, and $\frac{3}{4}$ are 3 times 3 or 9 twelfths. *Ans.* 9 twelfths.

Or thus: the given denominator 4, is contained in the required denominator 12, 3 times. Now, multiplying both terms of $\frac{3}{4}$ by 3, it becomes $\frac{9}{12}$, the fraction required. (Art. 35.)

NOTE.—It will be seen that this process consists in multiplying both terms of the given fraction by such a number as will make its denominator equal to the required denominator.

This number is easily found by dividing the *required* denominator by the *given* denominator.

2. Reduce $\frac{2}{3}$ to sixths.

3. A lad having 3 quarter dollars changed them for eighths: how many eighths did he have?

4. Reduce $\frac{2}{5}$ to tenths. To fifteenths. To hundredths.

5. Reduce $\frac{1}{6}$ to a fraction having 12 for its denominator.

6. Reduce $\frac{3}{7}$ to a fraction daving 14 for its denominator.

7. A man having $\frac{3}{4}$ of an acre of land, wished to divide it into lots of $\frac{1}{16}$ of an acre apiece: how many lots will he have?

8. Reduce $\frac{4}{7}$ to twenty-eighths. To thirty-fifths.

9. How many sixteenths in $\frac{5}{8}$? Twenty-fourths? Fortieths?

10. Reduce $\frac{7}{9}$ to twenty-sevenths. To forty-fifths.

11. How many thirtieths in $\frac{3}{5}$? Fiftieths?

12. Reduce $\frac{7}{12}$ to twenty-fourths. To sixtieths.

38. Reducing Mixed Numbers to Improper Fractions.

1. When a whole number and a fraction are written together, what is the expression called?

Ans. A ***Mixed Number.***

2. When the numerator of a fraction is *less* than the denominator, what?

Ans. A ***Proper Fraction.***

3. When the numerator is equal to or greater than the denominator, what?

Ans. An ***Improper Fraction.***

4. Reduce $6\frac{3}{4}$ pounds to fourths.

Analysis.—In 1 pound there are 4 fourths; and in 6 pounds, 6 times 4 or 24 fourths, and 3 fourths make 27 fourths, *Ans.*

5. In $5\frac{2}{3}$, how many thirds?

Reduce the following to improper fractions:

6. $5\frac{4}{7}$.	10. $6\frac{5}{9}$.	14. $10\frac{2}{7}$.
7. $8\frac{4}{5}$.	11. $8\frac{7}{10}$.	15. $12\frac{5}{8}$.
8. $7\frac{4}{3}$.	12. $11\frac{5}{12}$.	16. $20\frac{3}{5}$.
9. $8\frac{5}{4}$.	13. $13\frac{2}{3}$.	17. $50\frac{2}{3}$.

39. Reducing Improper Fractions to Whole or Mixed Numbers.

Ex. 1. In 9 half dollars, how many dollars?

Analysis.—Since there are 2 halves in one dollar, in 9 halves there are as many dollars as 2 is contained times in 9; and 2 is in 9, 4 times and 1 half over. Therefore, in 9 half dollars there are $4\frac{1}{2}$ dollars.

2. In 12 halves, how many units or ones?
3. Reduce $\frac{15}{3}$ to units.
4. Reduce $\frac{19}{4}$ to a mixed number.

Reduce the following to whole or mixed numbers:

5. $\frac{11}{3}$.	9. $\frac{19}{4}$.	13. $\frac{27}{4}$.	17. $\frac{64}{9}$.
6. $\frac{14}{7}$.	10. $\frac{21}{3}$.	14. $\frac{35}{8}$.	18. $\frac{59}{8}$.
7. $\frac{15}{4}$.	11. $\frac{28}{4}$.	15. $\frac{41}{10}$.	19. $\frac{75}{9}$.
8. $\frac{17}{7}$.	12. $\frac{31}{5}$.	16. $\frac{54}{9}$.	20. $\frac{100}{20}$.

40. Reducing Fractions to a Common Denominator.

Ex. 1. Reduce $\frac{3}{4}$ and $\frac{5}{8}$ to a common denominator.

Analysis.—The denominator 4 is contained in 8, the denominator of $\frac{5}{8}$, twice. Therefore, multiplying both terms of the fraction $\frac{3}{4}$ by 2, it becomes $\frac{6}{8}$; the given fractions now have a common denominator; $\frac{6}{8}$ and $\frac{5}{8}$, *Ans.*

Or, $\frac{3}{4}$ means $\frac{3}{4}$ of 1, and $1 = \frac{4}{4}$ or $\frac{8}{8}$. Now, $\frac{1}{4}$ of $\frac{8}{8}$ is $\frac{2}{8}$, and $\frac{3}{4}$ are 3 times $\frac{2}{8}$ or $\frac{6}{8}$. *Ans.* $\frac{6}{8}$ and $\frac{5}{8}$.

2. Reduce $\frac{2}{5}$ and $\frac{7}{10}$ to a common denominator.
3. Reduce $\frac{5}{7}$ and $\frac{8}{14}$ to a common denominator.
4. Reduce $\frac{3}{11}$ and $\frac{17}{33}$ to a common denominator.
5. Reduce $\frac{8}{12}$ and $\frac{1}{3}$ to a common denominator.

Analysis.—Reducing $\frac{8}{12}$ to thirds, we have $\frac{8}{12} = \frac{2}{3}$. *Ans.* $\frac{2}{3}$ and $\frac{1}{3}$.

6. Reduce $\frac{6}{8}$ and $\frac{9}{15}$ to a common denominator.
7. Reduce $\frac{2}{3}$ and $\frac{3}{4}$ to equivalent fractions having a common denominator.

Analysis.—The product of these denominators, $3 \times 4 = 12$; hence, 12 is a common multiple of them. (Art. 29, Note.)

Reducing the given fractions to twelfths, we have $\frac{2}{3} = \frac{8}{12}$, and $\frac{3}{4} = \frac{9}{12}$, *Ans.*

Or, thus: $1 = \frac{3}{3}$ or $\frac{12}{12}$. Now, $\frac{1}{3}$ of $\frac{12}{12}$ is $\frac{4}{12}$, and $\frac{2}{3}$ are $\frac{8}{12}$.

Again, $\frac{3}{4}$ mean $\frac{3}{4}$ of 1, and $1 = \frac{4}{4}$ or $\frac{12}{12}$. Now, $\frac{1}{4}$ of $\frac{12}{12}$ is $\frac{3}{12}$, and $\frac{3}{4}$ are $\frac{9}{12}$. Therefore, $\frac{8}{12}$ and $\frac{9}{12}$ are the fractions required.

8. Reduce $\frac{2}{3}$ and $\frac{3}{5}$ to a common denominator.
9. Reduce $\frac{3}{4}$ and $\frac{5}{6}$ to a common denominator.
10. Reduce $\frac{2}{7}$ and $\frac{3}{5}$ to a common denominator.
11. Reduce $\frac{5}{8}$ and $\frac{2}{7}$ to a common denominator.
12. Reduce $\frac{5}{4}$ and $\frac{4}{5}$ to a common denominator.
13. Reduce $\frac{5}{7}$ and $\frac{3}{4}$ to a common denominator.
14. Reduce $\frac{7}{8}$ and $\frac{3}{5}$ to a common denominator.
15. Reduce $\frac{5}{9}$ and $\frac{4}{7}$ to a common denominator.

ADDITION OF FRACTIONS.

41. Adding Fractions which have a Common Denominator.

Ex. 1. A man meeting some beggars, gave $\$\frac{5}{8}$ to one, $\$\frac{3}{8}$ to another, and $\$\frac{7}{8}$ to another: how much did he give to all?

Analysis.—Since these fractions have a *common denominator*, and refer to units of the same name, it follows that their numerators are *like fractional units*, viz.: *eighths*, and are added like simple units.

Now, 5 eighths and 3 eighths are 8 eighths, and 7 are 15 eighths, which are equal to $\$1\frac{7}{8}$, *Ans.*

2. A man sold 3 house lots, one containing $\frac{5}{10}$ of an acre, another $\frac{7}{10}$, and the other $\frac{9}{10}$ of an acre: how much did they all contain?

3. George gave $\$\frac{3}{4}$ to one companion, $\$\frac{2}{4}$ to another, $\$\frac{5}{4}$ to another: how many fourths did he give to all?

4. Paid $\$\frac{3}{8}$ for pears, $\$\frac{5}{8}$ for oranges, and $\$\frac{7}{8}$ for bananas: how much did the whole cost?

5. A lad worked 1 day for $\$\frac{3}{5}$, another day for $\$\frac{2}{5}$, and another for $\$\frac{7}{5}$: what did his wages amount to?

6. A lady paid $\$\frac{3}{12}$ for thread, $\$\frac{9}{12}$ for sewing silk, and $\$\frac{5}{12}$ for twist: how much did she pay for all?

7. Add $\frac{3}{7}$, $\frac{2}{7}$ and $\frac{5}{7}$.

8. Add $\frac{5}{12}$, $\frac{11}{12}$ and $\frac{8}{12}$.

9. Add $\frac{12}{19}$, $\frac{3}{19}$ and $\frac{7}{19}$.

10. Add $\frac{15}{37}$, $\frac{6}{37}$ and $\frac{19}{37}$.

11. A dairy woman sold 2 boxes of butter containing $12\frac{5}{8}$ pounds and $15\frac{7}{8}$ pounds respectively: how much butter did she sell?

Analysis.—15 pounds and 12 pounds are 27 pounds; $\frac{7}{8}$ and $\frac{5}{8}$ are $\frac{12}{8}$ equal $1\frac{4}{8}$, which added to 27 made $28\frac{4}{8}$ or $28\frac{1}{2}$ pounds. Therefore, etc.

12. How much are $\$13\frac{3}{7}$ and $\$10\frac{5}{7}$?

13. If a man pays $\$5\frac{1}{4}$ for a hat, and $\$9\frac{3}{4}$ for a vest, what will he pay for both?

14. What is the sum of $6\frac{5}{8}$ yards $+ 9\frac{3}{8}$ yards $+ 10\frac{7}{8}$ yards?

15. What is the sum of $8\frac{3}{16}$ pounds $+ 7\frac{5}{16}$ pounds $+ 12\frac{11}{16}$ pounds?

16. What is the sum of $\$25\frac{3}{8} + \$9\frac{1}{8} + \$7\frac{5}{8}$?

42. Adding Fractions which have Different Denominators.

Ex. 1. If a young man spends $\$\frac{2}{5}$ for a ride, and $\$\frac{3}{8}$ for a lunch, how much will he spend for both?

Analysis.—Since these fractions have *different* denominators, their numerators are *not* like fractional units, and cannot be added in their present form. We therefore reduce them to a common denominator, and then add them as above.

The product of the denominators $3 \times 5 = 15$; but $\$\frac{2}{5} = \$\frac{10}{15}$, and $\$\frac{3}{8} = \$\frac{9}{15}$. Now $\$\frac{10}{15} + \$\frac{9}{15} = \$\frac{19}{15}$, or $\$1\frac{4}{15}$, *Ans.* (Art. 29, Note.)

Or, thus: $\frac{2}{5}$ are equal to $\frac{10}{15}$, and $\frac{3}{8}$ are equal to $\frac{9}{15}$; and $\$\frac{10}{15} + \$\frac{9}{15} = \$\frac{19}{15}$, or $\$1\frac{4}{15}$, *Ans.*

2. A housekeeper paid $\$\frac{5}{6}$ for tea, $\$\frac{2}{12}$ for sugar: how much did she spend for both?

3. If a man mows $\frac{3}{4}$ of an acre in the morning, $\frac{5}{8}$ of an acre in the afternoon, how much will he mow during the day?

4. What is the sum of $\frac{5}{6}$ of a pound and $1\frac{1}{2}$ of a pound?

5. A general lost $\frac{1}{4}$ of his army by sickness, and $\frac{3}{8}$ by desertion: what part of his army did he lose?

6. A grocer sold $\frac{2}{10}$ of a chest of tea to one customer, and $\frac{4}{5}$ to another: what part of a chest did he sell?

7. If you pay $\$9\frac{1}{2}$ for a barrel of flour, and $\$3\frac{5}{8}$ for a barrel of apples, how much will both cost?

Analysis.—$9 and $3 are $12. Now, $\frac{1}{2} = \frac{4}{8}$, and $\frac{5}{8}$ plus $\frac{4}{8}$ are $\frac{9}{8} = \$1\frac{1}{8}$, which added to $12, make $\$13\frac{1}{8}$ or $\$13\frac{1}{8}$, *Ans.*

8. William gathered $7\frac{1}{4}$ quarts of chestnuts, and Harry $9\frac{5}{8}$ quarts: how many did both gather?

9. Frank paid $\$7\frac{5}{8}$ for his Latin Dictionary and $\$4\frac{1}{2}$ for his Virgil: what did he pay for both?

10. A farmer had two bins of corn; one containing $25\frac{3}{4}$ bushels, the other $20\frac{5}{12}$ bushels: how many bushels of corn had he?

11. Add $12\frac{1}{4}$ to $7\frac{1}{3}$.
12. Add $7\frac{2}{3}$ to $9\frac{7}{8}$.
13. Add $6\frac{1}{4}$ to $12\frac{1}{3}$.
14. Add $8\frac{3}{8}$ to $6\frac{3}{4}$.
15. Add $5\frac{1}{3}$ to $20\frac{5}{8}$.
16. Add $7\frac{1}{4}$ to $9\frac{7}{12}$.

17. Paid $\$8\frac{1}{2}$ for a barrel of flour, and $\$2\frac{1}{3}$ for a barrel of potatoes: what was the cost of both?

18. Paid $\$9\frac{2}{3}$ for a ton of coal, and $\$3\frac{1}{4}$ for a cord of wood: what did both cost?

19. If a pair of boots cost $\$7\frac{3}{8}$, and a pair of shoes $\$2\frac{1}{2}$, what will both cost?

20. If a coat cost $\$23\frac{1}{2}$, and a vest $\$10\frac{5}{6}$, what will both cost?

21. A man earned $\$8\frac{1}{3}$ per week above his expenses, which were $\$26\frac{2}{3}$: how much did he earn?

SUBTRACTION OF FRACTIONS.

43. Subtracting Fractions which have a Common Denominator.

Ex. 1. A man owning $\frac{11}{16}$ of a ship, sold $\frac{7}{16}$ of her: what part of the ship did he have left?

Analysis.—Since these fractions have a common denominator, their numerators are like fractional units, viz., sixteenths.

Now 7 sixteenths from 11 sixteenths leave $\frac{4}{16}$. Therefore he had $\frac{4}{16}$ or $\frac{1}{4}$ the ship left.

2. George had $\frac{4}{5}$ of a pineapple, and gave $\frac{2}{5}$ to his sister: how much had he left?

3. Henry having $\$\frac{9}{10}$, spent $\$\frac{2}{10}$ for a kite: how much had he left?

4. A man having $\frac{11}{12}$ of an acre of land, devoted $\frac{7}{12}$ to a vegetable garden, the balance to flowers: how large was his flower garden?

5. From $\frac{10}{17}$ take $\frac{6}{17}$.
6. From $\frac{17}{28}$ take $\frac{9}{28}$.
7. From $\frac{13}{22}$ take $\frac{8}{22}$.
8. From $\frac{31}{43}$ take $\frac{11}{43}$.

9. James having $\$10\frac{2}{5}$, paid $\$4\frac{3}{5}$ for a vest: how much had he left?

Analysis.—Since $\frac{3}{5}$, the fraction in the subtrahend, is larger than $\frac{2}{5}$, the fraction in the minuend, we take 1 from the minuend and reducing it to fifths, add it to $\frac{2}{5}$.

The minuend thus becomes $\$9\frac{7}{5}$. Now, $\frac{3}{5}$ from $\frac{7}{5}$ leave $\frac{4}{5}$, and $\$4$ from $\$9$ leave $\$5$, which united to $\frac{4}{5}$ make $\$5\frac{4}{5}$, *Ans.*

10. From $17\frac{1}{4}$ yards of silk, $5\frac{3}{4}$ yards have been cut: how many yards are left in the piece?

11. John has $\$20\frac{1}{2}$: if he spends $\$10\frac{1}{4}$, how much will he have left?

12. If you have $\$1$ and spend $\frac{3}{5}$ of it, how much will you have left?

Analysis.—$\$1 = \$\frac{5}{5}$; now $\$\frac{2}{5}$ from $\$\frac{5}{5}$ leaves $\$\frac{3}{5}$, *Ans.*

13. If you give to your sister $\frac{5}{8}$ of a melon, and keep the rest yourself, what part will you retain?

14. Augustus had a bushel of chestnuts, and the mice carried off $\frac{4}{5}$ of them: how many had he left?

15. The marked price of a hat is $\$3$, but for cash it can be bought for $\$\frac{3}{5}$ less: what sum will buy it?

16. From 7 bushels take $\frac{3}{4}$ bushel.
17. From 25 yards take $\frac{7}{8}$ yard.
18. From 15 pounds take $\frac{7}{16}$ pound.
19. From $\$25$ take $\$\frac{9}{10}$.
20. What is the difference between 100 cts. and $62\frac{1}{2}$ cts.?
21. What is the difference between $37\frac{1}{2}$ cts. and $87\frac{1}{2}$ cts.?
22. What is the difference between $6\frac{1}{4}$ cts. and $18\frac{3}{4}$ cts.?
23. What is the difference between $31\frac{1}{4}$ cts. and $\$1$?

44. Subtracting Fractions which have Different Denominators.

Ex. 1. A man bought $\frac{3}{4}$ of a barrel of apples, and on examination found $\frac{1}{3}$ of a barrel decayed: what part of a barrel did he have left?

Analysis.—Since these fractions have *different denominators*, one cannot be taken from the other in their present form.

Reducing them to a common denominator, $\frac{3}{4}$ becomes $\frac{9}{12}$, and $\frac{1}{3}$ becomes $\frac{4}{12}$. Now, $\frac{4}{12}$ from $\frac{9}{12}$ leaves $\frac{5}{12}$. *Ans.* $\frac{5}{12}$ bbl.

2. If you have $\$\frac{4}{5}$, and your brother has $\$\frac{7}{10}$, which has the greater sum?

3. A man has two gardens; one contains $\frac{4}{5}$ of an acre, the other, $\frac{5}{7}$: what is the difference between them?

4. From $\frac{2}{3}$ take $\frac{1}{4}$?
5. From $\frac{4}{5}$ take $\frac{2}{6}$.
6. From $\frac{5}{8}$ take $\frac{1}{3}$.
7. From $\frac{3}{4}$ take $\frac{2}{5}$.
8. From $\frac{3}{7}$ take $\frac{1}{2}$.
9. From $\frac{6}{7}$ take $\frac{3}{14}$.

10. A farmer employed two men, giving one $\$2\frac{1}{4}$, and the other $\$3\frac{1}{2}$ per day: what was the difference in their wages?

Analysis.—$\$3\frac{1}{2} = \$3\frac{2}{4}$. Now, $\$3\frac{2}{4}$ minus $\$2\frac{1}{4} = \$1\frac{1}{4}$.

11. A lady paid $\$12\frac{7}{8}$ for a hat and $\$8\frac{1}{4}$ for a pair of boots: what was the difference in the price?

12. If an orange costs $7\frac{1}{4}$ cts. and a melon $12\frac{1}{2}$ cts., what is the difference in price?

13. A citron is worth $12\frac{1}{2}$ cts. and a pineapple $18\frac{3}{4}$ cts.: what is the difference in their value?

14. If Horatio has $\$\frac{5}{8}$, and earns $\$\frac{3}{4}$ more, how much will he have left after spending $\$\frac{1}{3}$?

15. If you have $\$15\frac{7}{8}$ and spend $\$6\frac{1}{4}$ for a flute and $\$4\frac{1}{2}$ for a drum, how much money will you have left?

MULTIPLICATION OF FRACTIONS.

45. Multiplying a Fraction by a Whole Number.

Ex. 1. What cost 4 Readers, at $\$\frac{7}{12}$ apiece?

Analysis.—4 Readers will cost 4 times as much as 1 Reader, and 4 times $\$\frac{7}{12}$ are $\$\frac{28}{12}$, which are equal to $\$2\frac{4}{12}$ or $\$2\frac{1}{3}$. *Ans.* $\$2\frac{1}{3}$.

2. At $\frac{4}{5}$ of a cent apiece, what will 10 apples come to?

3. At $\$\frac{3}{4}$ a pound, what will 6 pounds nutmegs come to?

4. How many units in 6 times $\frac{12}{8}$?

5. How many units in 8 times $\frac{5}{10}$?

6. How many are 9 times $\frac{7}{12}$?

7. What cost 7 pair of slippers, at $\$\frac{5}{6}$ a pair?

8. If a barrel of flour will last a family of 5 persons $\frac{7}{8}$ of a month, how long will it last 1 person?

9. At $\$\frac{5}{8}$ a basket, what is the cost of 12 baskets of peaches?

10. What cost 10 dozen oranges, at $\$\frac{7}{12}$ a dozen?

11. If 7 men can mow a field in $\frac{9}{10}$ of a day, how long will it take 1 man to mow it?

12. How many loaves of bread will it take to supply 12 persons, allowing $\frac{6}{10}$ of a loaf to each?

13. Multiply $\frac{8}{15}$ by 4.

14. Multiply $\frac{5}{16}$ by 7.

15. Multiply $\frac{9}{17}$ by 6.

16. Multiply $\frac{6}{10}$ by 8.

46.

Ex. 1. At $6\frac{1}{4}$ cents each, what will 9 inkstands cost?

Analysis.—9 inkstands will cost 9 times as much as 1. Now 9 times 6 cts. are 54 cts., and 9 times $\frac{1}{4}$ cent are $\frac{9}{4}$, equal to $2\frac{1}{4}$ cts., which added to 54 cts. make $56\frac{1}{4}$ cts., *Ans.*

NOTE.—In multiplying a *mixed* by a *whole* number, it is advisable, in mental operations, to multiply the *integral* part first, then the *fraction*, and *unite* the results. (Art. 16, Note.)

2. What cost 7 doz. eggs, at $10\frac{1}{4}$ cts. a dozen?

3. At $12\frac{1}{2}$ cents apiece, what will 6 writing-books come to?

4. At $\$2\frac{1}{4}$ a pound, what will be the cost of 7 pounds of indigo?

5. If a man earns $\$9\frac{3}{4}$ per week, how much will he earn in 5 weeks?

6. What cost 10 sheep, at $\$3\frac{1}{4}$ a head?

7. A dairy-woman sent 4 boxes of butter to market, each containing $7\frac{1}{4}$ pounds: how many pounds did she send in all?

8. How many are 5 times $3\frac{2}{3}$? 6 times $4\frac{1}{4}$?

9. How many are 8 times $7\frac{1}{8}$? 9 times $5\frac{3}{8}$?

10. How many yards of cloth will it take to make 8 overcoats, allowing $4\frac{3}{4}$ yards to a coat?

11. What is the cost of 10 pounds of rice, at $8\frac{1}{3}$ cents a pound?

12. If a horse eats $2\frac{1}{2}$ bushels of oats in a week, how many bushels will he eat in 12 weeks?

13. What cost 11 barrels of cider, at $\$4\frac{3}{5}$ a barrel?

14. At $\$12\frac{1}{2}$ each, what will 6 flutes come to?

15. A has $\$7\frac{4}{5}$, and B has 5 times as much: how much money has B? How much have both?

16. What cost 6 dozen oranges, at $16\frac{2}{3}$ cents a dozen?

17. At $\$6\frac{1}{8}$ apiece, what will a dozen chairs cost?

18. If a man travels $8\frac{1}{2}$ miles an hour, how far will he travel in 7 hours?

19. What will 8 cords of wood cost, at $\$5\frac{1}{16}$ a cord?

20. At $\$11\frac{19}{20}$ a ton, what will 10 tons of coal cost?

21. Multiply $\$8\frac{5}{7}$ by 6.

22. Multiply $9\frac{3}{4}$ by 5.

23. Multiply $7\frac{3}{11}$ by 8.

24. Multiply $10\frac{1}{2}$ by 7.

25. What cost 12 hats, at $\$5\frac{3}{4}$ apiece?

26. How much must you pay for 10 melons, at $15\frac{1}{2}$ cents apiece?

27. A lady bought 12 yards of muslin, at $8\frac{1}{2}$ cents a yard: how much did it come to?

47. Multiplying a Whole Number by a Fraction.

Ex. 1. How is a whole number multiplied by a fraction?

Ans. By ***taking a certain portion*** of the multiplicand as many times, as there are *like portions* of a unit in the multiplier. Thus,

Multiplying by $\frac{1}{2}$, is taking 1 *half* of the multiplicand *once*. Thus, $6 \times \frac{1}{2} = 3$.

Multiplying by $\frac{1}{3}$, is taking 1 *third* of the multiplicand *once*. Thus, $6 \times \frac{1}{3} = 2$.

Multiplying by $\frac{2}{3}$ is taking 1 *third* of the multiplicand *twice*. Thus, $6 \times \frac{2}{3} = 4$.

2. What cost $\frac{1}{2}$ pound of tea, at 8 shillings a pound?

Analysis.—Since 1 pound costs 8 shillings, $\frac{1}{2}$ pound will cost $\frac{1}{2}$ of 8s.; and $\frac{1}{2}$ of 8s. is 4s., *Ans.*

3. What cost $\frac{1}{4}$ of a yard of silk, at 12 shillings a yard?

4. What is $\frac{1}{5}$ of 20? Of 35? Of 40?

5. What is the force of the word *of*, when placed between a *fraction* and a *whole* number, or between two *fractions*?

Ans. It has the force of the ***Sign of Multiplication*** $\times$.

Thus, $\frac{1}{3}$ of $9 = \frac{1}{3} \times 9$, $\frac{1}{4}$ of $\frac{1}{5} = \frac{1}{4} \times \frac{1}{5}$, etc.

6. What are fractions connected by the word *of*, called?

Ans. ***Compound Fractions.***

7. At \$15 a ton, what will $\frac{1}{6}$ of a ton of hay cost?

8. At \$17 a barrel, what cost $\frac{1}{8}$ of a barrel of beef?

9. What cost $\frac{1}{4}$ yard of poplin, at 22 shillings a yard?

10. What is $\frac{1}{8}$ of 25? $\frac{1}{9}$ of 30? $\frac{1}{10}$ of 41? $\frac{1}{12}$ of 54?

11. At \$16 a barrel, what cost $\frac{2}{3}$ of a barrel of molasses?

Analysis.—Since 3 thirds of a barrel cost \$16, 1 third of a barrel will cost 1 third of \$16, or \$5$\frac{1}{3}$, and $\frac{2}{3}$ will cost 2 times \$5$\frac{1}{3}$, or \$10$\frac{2}{3}$, *Ans.*

12. What cost $\frac{3}{4}$ of a pound of honey, at 32 cts. a pound?
13. What cost $\frac{4}{5}$ of a bushel of apples, at 40 cts. a bushel?
14. At 56 cents a yard, what will $\frac{5}{8}$ yard of ribbon cost?
15. What is $\frac{4}{5}$ of 30?
16. What is $\frac{6}{7}$ of 51?
17. What is $\frac{5}{7}$ of 42?
18. What is $\frac{9}{8}$ of 60?
19. At 20 shillings a day, how much will a man earn *in* $\frac{4}{5}$ of a day?
20. If a man can walk 30 miles in a day, how far can he walk in $\frac{7}{8}$ of a day?
21. If a basket of peaches is worth 75 cents, what are $\frac{5}{10}$ of a basket worth?

48. Ex. 1. What will $3\frac{1}{4}$ yards of edging cost, at 11 cents a yard?

Analysis.—If 1 yard costs 11 cents, $3\frac{1}{4}$ yards will cost $3\frac{1}{4}$ times 11 cts. Now 3 times 11 cts. are 33 cts., and $\frac{1}{4}$ of 11 cts. is $2\frac{3}{4}$ cts., which added to 33 cts. make $35\frac{3}{4}$ cents, *Ans.*

2. What cost $4\frac{1}{2}$ bushels of wheat, at 10 shillings?
3. What cost $6\frac{1}{4}$ pounds of cheese, at 12 cents a pound?
4. Bought $5\frac{2}{3}$ pounds of tea, at 6s. a pound: what did it come to?
5. At 9 dimes a yard, what cost $5\frac{3}{4}$ yds. of bombazine?
6. Multiply 8 by $6\frac{1}{4}$.
7. Multiply 8 by $5\frac{3}{4}$.
8. Multiply 11 by $6\frac{2}{3}$.
9. Multiply 6 by $4\frac{2}{3}$.
10. Multiply 10 by $6\frac{4}{5}$.
11. Multiply 12 by $8\frac{7}{10}$.
12. What cost $4\frac{2}{3}$ baskets of strawberries, at 6 cents a basket?
13. What cost $8\frac{2}{5}$ quarts of cherries, at 10 cents a quart?
14. At 8 dimes a bushel, what will $5\frac{3}{4}$ bu. of oats cost?
15. At 9 pence a pound, what cost $5\frac{3}{4}$ pounds of soda?
16. Multiply 9 by $3\frac{5}{8}$.
17. Multiply 10 by $5\frac{2}{3}$.
18. Multiply 11 by $6\frac{3}{4}$.
19. Multiply 11 by $12\frac{1}{4}$.
20. What cost $7\frac{1}{4}$ pounds of coffee, at 11 cents a pound?
21. What cost $5\frac{1}{2}$ dozen eggs, at 20 cents a dozen?

49. Multiplying a Fraction by a Fraction.

Ex. 1. What is the product of $\frac{3}{4}$ multiplied by $\frac{2}{5}$?

Analysis.—Multiplying by $\frac{2}{5}$ is taking $\frac{1}{5}$ of the multiplicand *twice.* (Art. 47.)

Now $\frac{3}{4} \times \frac{1}{5} = \frac{3}{20}$, and $\frac{2}{5}$ are 2 times $\frac{3}{20}$, or $\frac{6}{20}$, *Ans.* (Art. 34.)

NOTE.—1. It will be seen by inspection, that this analysis produces the same combinations and the same results, as multiplying the *two numerators* together for the numerator of the product, and the *two denominators* for the denominator of the product.

2. At $\$\frac{3}{4}$ a bushel, what will $\frac{3}{7}$ of a bushel of peas cost?

3. At $\$\frac{1}{2}$ a gallon, what will $\frac{3}{4}$ gallon of vinegar cost?

4. What will $\frac{3}{4}$ of a pound of powder cost, at $\$\frac{7}{8}$ a pound?

5. If a whole melon is worth $\$\frac{1}{4}$, what is $\frac{7}{8}$ of it worth?

6. Multiply $\$\frac{5}{7}$ by $\frac{2}{3}$.

7. Multiply $\frac{4}{5}$ pound by $\frac{5}{8}$.

8. Multiply $\frac{3}{8}$ yds. by $\frac{4}{5}$.

9. Multiply $\frac{7}{8}$ rod by $\frac{5}{9}$.

10. Reduce $\frac{3}{4}$ of $\frac{4}{5}$ to a simple fraction.

NOTE.—2. Compound Fractions are reduced to simple ones in the same manner as one fraction is multiplied by another.

Thus, $\frac{2}{4}$ of $\frac{3}{5} = \frac{3}{4} \times \frac{3}{5} = \frac{6}{20}$, the same as in Ex. 1.

11. Reduce $\frac{5}{8}$ of $\frac{9}{10}$ to a simple fraction.

12. A man owning a ship sold $\frac{3}{4}$ of $\frac{4}{5}$ of her: what part of the ship did he sell?

Reduce the following to simple fractions:

13. $\frac{2}{3}$ of $\frac{3}{8}$.
14. $\frac{1}{4}$ of $\frac{5}{8}$.
15. $\frac{2}{7}$ of $\frac{5}{7}$.
16. $\frac{2}{5}$ of $\frac{1}{4}$.
17. $\frac{3}{7}$ of $\frac{9}{10}$.
18. $\frac{4}{9}$ of $\frac{2}{3}$.
19. $\frac{5}{11}$ of $\frac{3}{4}$.
20. $\frac{7}{9}$ of $\frac{3}{8}$.
21. $\frac{2}{5}$ of $\frac{1}{3}$ of $\frac{3}{4}$.
22. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{9}$.
23. $\frac{3}{5}$ of $\frac{2}{7}$ of $\frac{5}{6}$.
24. $\frac{5}{8}$ of $\frac{4}{5}$ of $\frac{2}{7}$.

25. Reduce $\frac{3}{4}$ of $5\frac{1}{3}$ to a simple fraction.

Analysis.—$5\frac{1}{3} = \frac{16}{3}$; hence, $\frac{3}{4}$ of $5\frac{1}{3} = \frac{3}{4}$ of $\frac{16}{3}$, or $\frac{48}{12}$.

26. Reduce $\frac{3}{7}$ of $4\frac{2}{3}$ to a simple fraction.

27. Reduce $\frac{5}{8}$ of $\frac{1}{4}$ of $5\frac{1}{2}$ to a simple fraction.
28. Reduce $\frac{3}{10}$ of $\frac{1}{2}$ of $7\frac{1}{3}$ to a simple fraction.
29. What is $\frac{2}{3}$ of $\frac{4}{5}$ of a yard equal?
30. What is $\frac{4}{7}$ of $\frac{3}{4}$ of a mile equal?
31. What is the product of $\frac{2}{5}$ times $\frac{3}{8}$?
32. What is the product of $\frac{5}{7}$ times $\frac{3}{4}$?
33. What cost $\frac{3}{4}$ bushels of corn, at $\$\frac{4}{5}$ a bushel?

50. Ex. 1. What cost $3\frac{1}{3}$ bushels of wheat, at $\$1\frac{1}{4}$ a bushel?

Analysis.—Reducing the mixed numbers to fractions, $3\frac{1}{3}$ bu. $= \frac{10}{3}$ bu.; and $\$1\frac{1}{4} = \$\frac{5}{4}$.

Now as 1 bushel costs $\$\frac{5}{4}$, $\frac{10}{3}$ bushels will cost $\frac{10}{3}$ times $\$\frac{5}{4}$, or $\frac{10}{3}$ of $\$\frac{5}{4}$, and $\frac{10}{3}$ of $\$\frac{5}{4} = \$\frac{50}{12} = \$4\frac{2}{12}$ or $\$4\frac{1}{6}$, *Ans.*

2. What cost $2\frac{1}{2}$ bushels of salt, at $1\frac{1}{4}$ dime a bushel?
3. If a man earns $\$1\frac{1}{2}$ a day, how much can he earn in $5\frac{3}{4}$ days?
4. At $\$2\frac{1}{5}$ a pound, what will $3\frac{1}{4}$ pounds of spice cost?
5. What cost $5\frac{1}{2}$ pounds of venison, at $3\frac{1}{4}$ dimes a pound?
6. What is $3\frac{1}{3}$ times $5\frac{1}{2}$?
7. What is $4\frac{1}{5}$ times $3\frac{1}{2}$?
8. What is $2\frac{3}{4}$ times $4\frac{1}{2}$?
9. What is $2\frac{2}{3}$ times $3\frac{2}{4}$?
10. If a man travels $3\frac{1}{4}$ miles an hour, how far will he go in $2\frac{1}{2}$ hours?
11. A clerk sold $2\frac{1}{2}$ yards of ribbon, at $3\frac{3}{4}$ dimes a yard: how much did it come to?
12. What is the product of $3\frac{1}{2}$ times $2\frac{1}{4}$?
13. What is the product of $2\frac{7}{8}$ times $5\frac{1}{2}$?
14. What is the product of $3\frac{1}{2}$ times $2\frac{2}{8}$?
15. What will $5\frac{1}{2}$ yards of satin brocatelle come to, at $\$8\frac{3}{4}$ a yard?
16. When butter is $33\frac{1}{3}$ cents a pound, what will $4\frac{1}{2}$ pounds come to?

DIVISION OF FRACTIONS.

51. Dividing a Fraction by a Whole Number.

Ex. 1. If 3 pineapples cost $\$\frac{6}{8}$, what will 1 pineapple cost?

Analysis.—1 is $\frac{1}{3}$ of 3; hence, 1 pine will cost $\frac{1}{3}$ of $\$\frac{6}{8}$, and 1 third of $\$\frac{6}{8}$ is $\frac{2}{8}$ or $\$\frac{1}{4}$. Therefore, 1 pine will cost $\$\frac{1}{4}$.

Or, thus: If 3 pines cost $\$\frac{6}{8}$, 1 pine will cost $\frac{1}{3}$ of $\$\frac{6}{8}$, which is $\frac{2}{8}$ or $\$\frac{1}{4}$, *Ans.*

2. If you pay $\$\frac{4}{5}$ for 2 slates, what is that for each?

3. Henry gave $\$\frac{9}{10}$ for 3 kites: what did each cost?

4. If $\frac{9}{12}$ of an acre of land is divided into 5 house-lots, how much land will each lot contain?

5. Divide $\frac{8}{10}$ by 4.
6. Divide $\frac{6}{25}$ by 8.
7. Divide $\frac{45}{16}$ by 9.
8. Divide $\frac{12}{15}$ by 6.
9. Divide $\frac{21}{35}$ by 7.
10. Divide $\frac{48}{27}$ by 12.

11. William paid $\$\frac{35}{8}$ for 7 rakes: how much was that apiece?

52. Ex. 1. If 4 melons cost $\$\frac{7}{10}$, what will 1 melon cost?

Analysis.—1 melon is $\frac{1}{4}$ of 4 melons; therefore 1 melon will cost $\frac{1}{4}$ of $\$\frac{7}{10}$; and $\frac{1}{4}$ of $\$\frac{7}{10}$ is $\$\frac{7}{40}$, *Ans.*

Or thus: since the numerator 7 cannot be divided by 4 without a remainder, we multiply the denominator 10 by it, and obtain $\$\frac{7}{40}$, the same as before. (Art. 34.)

2. If 2 pears cost $\frac{3}{8}$ of a dime, what will 1 pear cost?

3. What is the cost of 1 bushel of oats, at the rate of 3 bushels for $\$\frac{7}{8}$?

4. If $\frac{1}{2}$ a ton of coal is divided equally into 5 parts, what part of a ton is one of these parts?

5. Divide $\frac{3}{7}$ by 6.
6. Divide $\frac{11}{3}$ by 8.
7. Divide $\frac{12}{7}$ by 11.
8. Divide $\frac{5}{8}$ by 7.
9. Divide $\frac{13}{5}$ by 9.
10. Divide $\frac{23}{9}$ by 12.

11. George having 8 quarts of chestnuts, sold $\frac{1}{2}$ of them for $\$\frac{3}{5}$: what part of a dollar was that a quart?

12. If a family of 7 persons use $\frac{5}{6}$ of a barrel of flour in a month, how much will 1 person use in the same time?

13. Bought 5 quarts of gooseberries for $\$\frac{3}{4}$: what was that a quart?

14. William picked 12 quarts of blackberries, and sold $\frac{1}{3}$ of them for $\$\frac{2}{3}$: how much did he receive a quart?

53. Ex. 1. Bought 6 bushels of apples for $\$4\frac{1}{2}$: what was that per bushel?

Analysis.—Since 6 bushels cost $\$4\frac{1}{2}$, 1 bushel cost $\frac{1}{6}$ of $\$4\frac{1}{2}$. But $\$4\frac{1}{2} = \$\frac{9}{2}$, or $\$\frac{54}{12}$, and 1 sixth of $\$\frac{54}{12}$ is $\$\frac{9}{12}$ or $\$\frac{3}{4}$. *Ans.*

2. If 2 pounds of tea cost $\$2\frac{1}{4}$, what will 1 pound cost?

3. If 4 baskets of peaches cost $\$5\frac{1}{2}$, what will 1 basket cost?

4. Sold 3 barrels of flour for $\$20\frac{1}{2}$: what was that a barrel?

5. How many times is 6 contained in $14\frac{1}{2}$?

6. Bought 6 cocoanuts for $\$2\frac{3}{4}$: what was the price of each?

7. If I pay $\$12\frac{1}{2}$ for 5 sheep, what is the cost of each?

8. Divide $7\frac{1}{2}$ by 6.
9. Divide $7\frac{3}{4}$ by 10.
10. Divide $8\frac{4}{5}$ by 7.
11. Divide $6\frac{1}{4}$ by 10.
12. Divide $16\frac{2}{3}$ by 12.
13. Divide $20\frac{1}{4}$ by 15.

54. Dividing a Whole Number by a Fraction.

Ex. 1. How many times is $\frac{1}{4}$ contained in 5?

Analysis.—In 1 unit there are 4 fourths, and in 5, there are 5 times 4, or 20 fourths. Hence, $\frac{1}{4}$ is contained in 5, 20 times.

NOTE.—Examples like the above, whose numerator is 1, are equivalent to this: "5 is equal to how many fourths?"

They may be answered thus: "5 is equal to 5 times 4 or 20 fourths."

2. How many bananas can be bought for 6 dimes, at $\frac{1}{2}$ dime a piece?

3. A baker distributed 6 pies among a group of children, giving $\frac{1}{4}$ of a pie to each: how many were there in the group?

4. A man having 10 acres of land, divided it into house-lots, each containing $\frac{1}{8}$ of an acre: how many lots did he make?

5. If you lay up $\$\frac{1}{10}$ each day, how long will it take you to lay up \$10?

6. Seven is equal to how many fourths?

7. Ten is equal to how many fifths?

8. Twelve is equal to how many sevenths?

55. Ex. 1. At $\$\frac{3}{4}$ a gallon, how many gallons of milk can you buy for \$5?

Analysis.—In \$5 there are $\$\frac{20}{4}$. Now if $\$\frac{3}{4}$ will buy 1 gallon, $\$\frac{20}{4}$ will buy as many gallons as 3 is contained times in 20, which are $6\frac{2}{3}$. Therefore, etc.

Or, thus: In 5 there are 20 fourths, and 3 fourths are contained in 20 fourths, $6\frac{2}{3}$ times. *Ans.* $6\frac{2}{3}$ gallons.

NOTE.—The *second* is the same as reducing the whole number to a fraction having the *same denominator* as that of the given fraction, then dividing its numerator by the numerator of the fraction.

2. At $\frac{2}{3}$ of a dime apiece, how many oranges can you buy for 6 dimes?

3. If you spend $\$\frac{2}{3}$ per day, how long will it take you to spend \$12?

4. How is an integer divided by a fraction?

5. How many yards of flannel, at $\$\frac{3}{4}$ a yard, can you buy for \$8?

6. Divide 7 by $\frac{3}{5}$.
7. Divide 6 by $\frac{7}{8}$.
8. Divide 9 by $\frac{5}{6}$.
9. Divide 11 by $\frac{4}{7}$.
10. Divide 10 by $\frac{4}{5}$.
11. Divide 11 by $\frac{7}{8}$.
12. Divide 12 by $\frac{9}{5}$.
13. Divide 9 by $\frac{7}{8}$.

56. Ex. 1. At $6\frac{1}{4}$ cents apiece, how many lemons will 20 cents buy?

Analysis.—$6\frac{1}{4} = \frac{25}{4}$; the question is: how many times are $\frac{25}{4}$ contained in 20. Now 20 equals $\frac{80}{4}$, and $\frac{80}{4} \div \frac{25}{4} = 3\frac{5}{25}$ or $3\frac{1}{5}$.

2. How many slates at $1\frac{1}{2}$ dime apiece, can be bought for 6 dimes?

3. If pears are $2\frac{1}{2}$ cents each, how many can you buy for 12 cents?

4. If braid is $3\frac{1}{4}$ cents a yard, how many yards can you buy for 8 cents?

5. How many times are $4\frac{2}{3}$ contained in 14?

6. At $12\frac{1}{2}$ cents a yard, how many yards of muslin can be had for 30 cents?

7. Divide 15 by $6\frac{1}{4}$.
8. Divide 21 by $5\frac{1}{4}$.
9. Divide 30 by $2\frac{1}{2}$.
10. Divide 50 by $6\frac{2}{3}$.

57. Dividing a Fraction by a Fraction when they have a Common Denominator.

Ex. 1. How many Grammars at $\$\frac{2}{5}$ apiece, can you buy for $\$\frac{8}{5}$?

Analysis.—If $\$\frac{2}{5}$ will buy 1 Grammar, $\$\frac{8}{5}$ will buy as many Grammars as $\$\frac{2}{5}$ are contained times in $\frac{8}{5}$; and 2 fifths are in 8 fifths, 4 times. *Ans.* 4 Grammars.

2. At $\frac{1}{4}$ of a dime apiece, how many pears can you purchase for $\frac{3}{4}$ of a dime?

3. How long will $\frac{6}{8}$ of a barrel of sugar last a family, if they use $\frac{3}{8}$ of a barrel a month?

4. How many times are $\frac{3}{12}$ contained in $\frac{9}{12}$?

5. How many times are $\frac{5}{17}$ contained in $\frac{40}{17}$?

6. At $\frac{5}{20}$ of a pound sterling apiece, how many caps can I get for $\frac{15}{20}$ of a pound?

7. If you can buy a pound of tea for $\$\frac{4}{5}$, how many pounds can you buy for $\$\frac{12}{5}$?

58. Dividing a Fraction by a Fraction when they have Different Denominators.

Ex. 1. How much cambric, at $\$\frac{3}{4}$ a yard, can you buy with $\$\frac{5}{6}$?

Analysis.—Since these fractions have *different* denominators they express *unlike* fractional units; therefore, one cannot be divided by the other in their present form.

Reducing them to a common denominator, we have $\$\frac{3}{4} = \$\frac{9}{12}$, and $\$\frac{5}{6} = \$\frac{10}{12}$. (Art. 40.)

Now 9 twelfths are contained in 10 twelfths $1\frac{1}{9}$ times. Therefore, you can buy $1\frac{1}{9}$ yard, *Ans.*

2. How much cinnamon, at $\$\frac{2}{3}$ a pound, can be obtained for $\$\frac{5}{6}$?

3. At $\frac{2}{3}$ of a dime apiece, how many peaches can be bought for $\frac{8}{10}$ of a dime?

4. How many times are $\frac{3}{7}$ contained in $1\frac{1}{4}$?

5. How many building lots, each containing $\frac{2}{12}$ of an acre, can be made of $\frac{6}{8}$ of an acre?

6. When butter is $\$\frac{3}{10}$ a pound, how much can you get for $\$\frac{4}{5}$?

7. If a horse eats $\frac{1}{3}$ of a ton of hay each month, how long will $\frac{9}{10}$ of a ton last him?

8. How many times are $\frac{3}{8}$ contained in $\frac{5}{6}$?

9. How many times are $\frac{6}{5}$ contained in $1\frac{9}{2}$?

10. If $\frac{2}{3}$ of a pound of coffee will last a family a week, how long will $\frac{9}{10}$ of a pound last them?

11. Into how many parts can you divide $\frac{9}{10}$ of a pie, that each part may contain $\frac{3}{20}$ of the pie?

12. How many times are $\frac{2}{6}$ contained in $\frac{5}{12}$?

13. How many times are $\frac{3}{7}$ contained in $\frac{9}{10}$?

14. How many times is $\frac{1}{9}$ contained in $\frac{7}{8}$?

15. How many times are $\frac{2}{10}$ contained in $1\frac{9}{20}$?

16. A lad having $\$\frac{8}{9}$, laid it out in chestnuts, at $\$\frac{2}{3}$ a peck: how many pecks did he buy?

59. Ex. 1. At $2\frac{1}{2}$ cents apiece, how many pears can be bought for $6\frac{1}{4}$ cents?

Analysis.—If $2\frac{1}{2}$ cents will buy 1 pear, $6\frac{1}{4}$ cents will buy as many as there are $2\frac{1}{2}$ cents in $6\frac{1}{4}$ cents.

But $2\frac{1}{2} = \frac{5}{2}$ or $\frac{10}{4}$ cts., and $6\frac{1}{4}$ cts. $= \frac{25}{4}$ cts. Now 10 fourths are contained in 25 fourths $2\frac{1}{2}$ times. Therefore, etc.

2 How many gallons of milk, at $3\frac{1}{2}$ dimes a gallon, can be bought with $10\frac{1}{2}$ dimes?

3. At $\$2\frac{1}{4}$ a cord, how much wood can be purchased with $\$7\frac{1}{2}$?

4. How many times are $3\frac{1}{3}$ contained in $8\frac{2}{3}$?

5. How many times are $1\frac{2}{5}$ contained in $4\frac{2}{10}$?

6. How long will $16\frac{1}{4}$ ounces of snuff last a person, if he takes $3\frac{1}{2}$ ounces a month?

7. At $2\frac{1}{5}$ dimes a pound, how much ginger can be had for $6\frac{3}{10}$ dimes?

8. How many times are $2\frac{2}{3}$ contained in $8\frac{1}{2}$?

9. How many times are $3\frac{1}{3}$ contained in $7\frac{1}{2}$?

10. At $\$1\frac{3}{7}$ a pound, how much tea can be bought for $\$11\frac{3}{7}$?

11. How many peaches, at $2\frac{1}{2}$ cents, can be bought for $12\frac{1}{2}$ cents?

12. How many times are $3\frac{1}{4}$ contained in $7\frac{9}{12}$?

13. At $\$1\frac{1}{8}$ a yard, how many yards of satinet can be had for $\$10\frac{1}{2}$?

14. If $3\frac{1}{3}$ pounds of tea last a family a month, how long will $25\frac{2}{3}$ pounds last them?

15. When $\$4\frac{1}{2}$ will buy a barrel of flour, how much can be bought for $\$37\frac{1}{2}$?

16. How many times are $3\frac{1}{4}$ contained in $7\frac{1}{2}$?

17. How many times are $7\frac{1}{2}$ contained in $15\frac{1}{4}$?

18. If you pay $6\frac{1}{4}$ cents a mile, how far can you ride for $31\frac{1}{4}$ cents?

19. Paid $\$33\frac{1}{3}$ for $8\frac{1}{3}$ yards of tarlatan: what was that per yard?

CHAPTER VII.

FRACTIONAL RELATION OF NUMBERS.

60. Finding what part one number is of another.

That numbers may be compared with each other, they must be so far of the *same nature* that one may properly be said to be a *part* of the other.

Thus, an *inch* may be compared with a *foot*; for *one* is a *twelfth part* of the other. But it cannot be said that a *foot* is any part of an *hour*; therefore the *former* cannot be compared with the *latter*.

Ex. 1. What part of 3 dollars is 1 dollar?

Analysis.—3 is 3 times 1. Therefore \$1 is $\frac{1}{3}$ of \$3.

2. What part of 3 is 2?

Analysis.—Since 1 is $\frac{1}{3}$ of 3, 2 must be 2 times $\frac{1}{3}$ of 3 or $\frac{2}{3}$ of 3 *Ans.*

3. What part of 3 is 4? *Ans* $\frac{4}{3}$.

NOTE.—In finding what part one number is of another, make the number called the *part*, whether it be smaller or larger, the *numerator*, and the other number the *denominator*.

The denominator is also known by having the word *of* before it. The fraction thus formed will be the answer required.

4. What part of 7 yards is 5 yards?

5. What part of 8 is 1? Is 2? 3? 5? 9? 10?

6. If a barrel of flour is equally divided among 5 persons, what part of it will 2 persons receive?

7. What part of 6 roses are 4 roses? 5? 7? 13? 15?

8. What part of a week is 5 days? 6 days? 10 days?

9. When coal is \$9 a ton, how much will \$2 buy?

10. What part of 11 is 5? 3? 6? 8? 9? 13?

11. John is 11 years old and William is 8 years: what part of John's age is William's?

12. What part of 14 is 5? 4? 7? 6? 8? 9? 10? 11?

13. A milkmaid having 12 quarts of milk, spilt 7 quarts: what part did she lose? What part did she have left?

14. Ten is what part of 13? Of 17? Of 11? Of 27?

15. If you can buy a gallon of vinegar for 15 cents, what part of a gallon can you buy for 3 cents? For 4 cents? 7 cents? 11 cents? 15 cents? 17 cents?

16. When land is worth $20 an acre, what part of an acre can you buy for $3? For $5? $7? $11? $17? $20? $25?

17. £19 is what part of £23? Of £31? Of £60? Of £100?

18. If you divide 16 peaches equally among 3 children, what part, and how many peaches will each receive?

19. A market-woman having 7 dozen eggs, let them fall and broke 3 dozen: what part did she break? What part, and how many did she have left?

20. A farmer having 30 acres of land, fenced it into 3 equal pastures: what part of his land, and how many acres did 2 of his pastures contain?

21. What part of 24 is $\frac{1}{5}$ of 15?

22. What part of 36 is $\frac{2}{3}$ of 24?

23. 3 times 4 is what part of 29?

24. 7 times 3 is what part of 46?

25. What part of 81 is $\frac{1}{7}$ of 28?

26. What part of 42 is $\frac{3}{4}$ of 20?

27. Samuel having 50 cents, paid $\frac{2}{5}$ of it for a pair of skates: how much, and what part of his money did he give for his skates?

28. 4 times 7 is what part of 5 times 9?

29. 6 times 8 is what part of 7 times 9?

30. 1 ninth of 72 is what part of 7 times 4?

61. Finding what part one *Fraction* is of another.

Ex. 1. What part of $\frac{2}{3}$ is $\frac{1}{3}$?

Analysis.—1 is 1 half of 2. Therefore, 1 third must be $\frac{1}{2}$ of 2 thirds?

Note.—1. When fractions have a *common* denominator, their *numerators* are compared as whole numbers; for, they are *like parts* of a unit.

2. If you can buy a bushel of corn for $\$\frac{3}{4}$, what part of a bushel can you buy for $\$\frac{1}{4}$?

3. What part of $\frac{3}{5}$ is $\frac{2}{5}$?

4. What part of $\frac{4}{7}$ is $\frac{3}{7}$?

5. What part of $\frac{7}{8}$ is $\frac{3}{8}$?

6. What part of $\frac{3}{4}$ is $\frac{2}{3}$?

Analysis.—$\frac{2}{3} = \frac{9}{12}$, and $\frac{3}{4} = \frac{8}{12}$; and 8 twelfths are $\frac{8}{9}$ of 9 twelfths. *Ans.* $\frac{8}{9}$.

Note.—2. The given fractions must always be reduced to a *common* denominator, then we may compare them as above.

7. What part of $\frac{3}{5}$ is $\frac{4}{10}$?

8. What part of $\frac{7}{8}$ is $\frac{3}{4}$?

9. What part of $\frac{5}{12}$ is $\frac{2}{3}$?

10. If you can buy 1 yard of cloth for $£\frac{7}{10}$, what part of a yard can you buy for $£\frac{3}{20}$?

11. What part of 6 is $3\frac{1}{2}$?

Analysis.—$6 = \frac{12}{2}$, and $3\frac{1}{2} = \frac{7}{2}$; and 7 halves are $\frac{7}{12}$ of 12 halves. *Ans.* $\frac{7}{12}$.

12. What part of 10 is $3\frac{2}{3}$?

13. What part of 12 is $2\frac{3}{4}$?

14. What part of 20 is $4\frac{3}{5}$?

15. What part of $4\frac{1}{2}$ is 3?

16. What part of $8\frac{2}{3}$ is 5?

17. What part of $6\frac{1}{4}$ is $2\frac{1}{2}$?

18. What part of $10\frac{2}{3}$ is $4\frac{3}{5}$?

19. A man having $\$20\frac{1}{2}$, spent $\$7\frac{1}{2}$: what part of his money did he spend?

20. A schoolboy having 24 examples in a lesson, solved $\frac{2}{3}$ of $\frac{5}{8}$ of them in the forenoon: what part of them did he solve in the afternoon?

21. Henry's fish-line was $15\frac{1}{2}$ feet long; by accident he lost $3\frac{1}{2}$ feet: what part of his line did he lose?

22. How do you find what part one fraction is of another, when they have a common denominator? How when they have not?

62. Finding a number, when a fractional part of it is given.

Ex. 1. 4 is 1 third of what number?

Analysis.—Since 4 is 1 third of the number, 3 thirds must be 3 times 4 or 12. Therefore, 4 is $\frac{1}{3}$ of 12.

Or, thus: 4 is a third of 3 times 4, which are 12.

2. 7 is a fourth of what number?

3. 8 is $\frac{1}{2}$ of what number? $\frac{1}{4}$ of what? $\frac{1}{5}$? $\frac{1}{7}$?

4. 9 is $\frac{1}{3}$ of what number? $\frac{1}{6}$ of what? $\frac{1}{8}$? $\frac{1}{10}$? $\frac{1}{11}$?

5. 24 is $\frac{4}{5}$ of what number?

Analysis.—Since 24 is $\frac{4}{5}$ of the number, $\frac{1}{5}$ of it must be $\frac{1}{4}$ of 24, which is 6, and $\frac{5}{5}$ are 3 times 6, or 18, *Ans.*

NOTE.—The learner should observe the difference between finding a *fractional part* of a given number, and finding a *number* when a fractional part of it is given. (Art. 31.)

In the *former*, we divide by the *denominator* of the fraction; in the *latter*, by the *numerator*. If the pupil is at a loss which to take for the *divisor*, let him substitute the word *parts* for the denominator.

6. 50 is $\frac{2}{3}$ of what number? $\frac{4}{5}$ of what?

7. 75 is $\frac{3}{4}$ of what number? $\frac{5}{4}$ of what?

8. 100 is $\frac{2}{3}$ of what number? $\frac{5}{6}$ of what?

9. 200 is $\frac{4}{5}$ of what number? $\frac{2}{5}$ of what?

10. 300 is $\frac{1}{2}$ of what number? $\frac{3}{4}$ of what?

11. 63 is $\frac{7}{8}$ of what number? $\frac{9}{8}$ of what?

12. 54 is $\frac{9}{4}$ of what number? $\frac{6}{4}$ of what?

13. 96 is $\frac{8}{3}$ of what number? $\frac{12}{3}$ of what?

63. Ex. 1. 44 is $5\frac{1}{2}$ times what number?

Analysis.—$5\frac{1}{2}$ is equal to $\frac{11}{2}$. The question then is this: 44 is $\frac{11}{2}$ of what number. Now if 44 is $\frac{11}{2}$ of a number, $\frac{1}{2}$ is $\frac{1}{11}$ of 44, which is 4, and $\frac{2}{2}$ are 2 times 4, or 8, *Ans.*

Or, thus: 44 is the product of two factors, one of which is $5\frac{1}{2}$; hence, the other factor must be $44 \div 5\frac{1}{2}$; and $44 \div 5\frac{1}{2} = 8$, the same as above. (Art. 26.)

2. 60 is $6\frac{2}{3}$ times what number?

3. 100 is $8\frac{1}{3}$ times what number?

4. 150 is $7\frac{1}{7}$ times what number?

5. $\frac{2}{3}$ of 24 is $\frac{1}{4}$ of what number?

Analysis.—$\frac{1}{4}$ of 24 is 8, and 2 thirds are 2 times 8, or 16. Now 16 is $\frac{1}{4}$ of 4 times 16, which are 64. Therefore, etc.

6. $\frac{2}{5}$ of 40 are $\frac{1}{3}$ of what number?

7. $\frac{1}{3}$ of 60 are $\frac{3}{5}$ of what number?

8. $\frac{5}{7}$ of 49 are $\frac{1}{2}$ of what number?

9. $\frac{12}{9}$ of 36 are $\frac{4}{5}$ of what number?

10. If $\frac{3}{4}$ of a ton of hay cost $15, what will a ton cost?

11. A builder paid $20 for $\frac{5}{8}$ of an acre of land: what was that per acre?

12. A man paid 21 shillings for $\frac{7}{10}$ of a barrel of flour: how much was that a barrel?

13. If you pay 44 cents for $\frac{4}{5}$ of a bushel of apples, what is that a bushel?

14. A boy being asked how many pears he had, replied that he had 50 apples, which was $\frac{2}{3}$ the number of his pears: how many pears had he?

15. Henry lost 42 yards of his kite line, which was $\frac{6}{7}$ of his whole line: what was its length?

16. Four times $\frac{3}{8}$ of $32 is $\frac{4}{7}$ of the price a man paid for a cow: what did she cost him?

17. Seven times $\frac{2}{7}$ of 28, are $\frac{8}{11}$ of what number?

18. Five times $\frac{2}{3}$ of 18 cents are $\frac{9}{10}$ of the price George paid for an Arithmetic: what did it cost him?

64. Ex. 1. 36 is $\frac{4}{5}$ of how many times 8?

Analysis.—This example involves two inquiries: 1st. 36 is $\frac{4}{5}$ of what number? 2d. How many times is 8 contained in that number?

Since 36 is $\frac{4}{5}$ of a certain number, $\frac{1}{5}$ is $\frac{1}{4}$ of 36, which is 9, and $\frac{5}{5}$, are 5 times 9, or 45. Again, 8 is contained in 45, $5\frac{5}{8}$ times. Therefore, 36 is $\frac{4}{5}$ of $5\frac{5}{8}$ times 8.

2. 42 is $\frac{6}{7}$ of how many times 7?

3. 30 is $\frac{5}{8}$ of how many times 9?

4. 45 is $\frac{5}{6}$ of how many times 8?

5. 56 is $\frac{7}{9}$ of how many times 10?

6. 63 is $\frac{9}{10}$ of how many times 11?

7. Henry spent 42 cents for a ball, which was $\frac{6}{7}$ of the money he still had; he then laid out the rest of his money in oranges at 7 cents apiece: how many oranges did he buy?

8. A grocer sold 45 lbs. of butter, which was equal to $\frac{3}{7}$ of what he had left. How many jars would it take to hold the remainder, if he put 9 lbs. in a jar?

65. Ex. 1. 24 is $\frac{3}{5}$ of how many times $\frac{3}{4}$ of 12?

Analysis.—This example involves three inquiries: 1st. 24 is $\frac{3}{5}$ of what number? 2d. What is $\frac{3}{4}$ of 12? 3d. How many times does the former contain the latter?

Since 24 is $\frac{3}{5}$ of a number, $\frac{1}{5}$ is $\frac{1}{3}$ of 24, which is 8, and $\frac{5}{5}$ are 5 times 8, which are 40. Next, $\frac{1}{4}$ of 12 is 3, and $\frac{3}{4}$ are 3 times 3, or 9. Finally, 9 is contained in 40, $4\frac{4}{9}$ times. Therefore, 24 is $\frac{3}{5}$ of $4\frac{4}{9}$ times $\frac{3}{4}$ of 12.

NOTE.—If this and the subsequent combinations in this Chapter are deemed too difficult for the class, they may be omitted till review.

2. 30 is $\frac{5}{7}$ of how many times $\frac{2}{3}$ of 18?

3. 48 is $\frac{6}{10}$ of how many times $\frac{4}{5}$ of 25?

4. 64 is $\frac{8}{11}$ of how many times $\frac{5}{6}$ of 42?

5. 45 is $\frac{5}{7}$ of how many times $\frac{2}{5}$ of 30?

6. 56 is $\frac{7}{10}$ of how many times $\frac{3}{11}$ of 44?

7. A grocer paid $28 for a hogshead of molasses, which was $\frac{7}{8}$ of what he sold it for; he took his pay in flour, at $\frac{3}{5}$ of $12 a barrel: how much flour did he receive?

8. A man paid $50 for a wagon, and the sum he paid was $\frac{5}{8}$ of what he sold it for; he took his pay in ploughs, at $\frac{5}{7}$ of $14 apiece: how many ploughs did he receive for his wagon?

66. Ex. 1. $\frac{3}{4}$ of 36 are $\frac{2}{3}$ of how many times 7?

Analysis.—First, $\frac{1}{4}$ of 36 is 9, and $\frac{3}{4}$ are 3 times 9 or 27. Next, if 27 is $\frac{2}{3}$ of a certain number, $\frac{1}{3}$ is $\frac{1}{2}$ of 27, which is 9, and $\frac{3}{3}$ are 5 times 9, or 45. Finally, 7 is contained in 45, $6\frac{3}{7}$ times. Therefore, $\frac{3}{4}$ of 36 are $\frac{2}{3}$ of $6\frac{3}{7}$ times 7.

2. $\frac{2}{3}$ of 27 are $\frac{6}{7}$ of how many times 8?
3. $\frac{4}{5}$ of 30 are $\frac{4}{5}$ of how many times 10?
4. $\frac{2}{7}$ of 42 are $\frac{2}{3}$ of how many times 5?
5. $\frac{5}{8}$ of 56 are $\frac{7}{5}$ of how many times 11?
6. $\frac{7}{9}$ of 72 are $\frac{8}{10}$ of many times 12?

67. Ex. 1. $\frac{6}{7}$ of 42 are 3 times $\frac{2}{3}$ of what number?

Analysis.—First, $\frac{1}{7}$ of 42 is 6, and $\frac{6}{7}$ are 6 times 6 or 36. Next, 3 times $\frac{2}{3}$ are $\frac{6}{3}$. Now if 36 is $\frac{6}{3}$ of a certain number, $\frac{1}{3}$ must be $\frac{1}{6}$ of 36, which is 6, and $\frac{3}{3}$ must be 5 times 6, or 30, *Ans.*

2. $\frac{4}{5}$ of 60 are 4 times $\frac{3}{10}$ of what number?
3. $\frac{6}{8}$ of 72 are 6 times $\frac{1}{8}$ of what number?
4. $\frac{4}{7}$ of 63 are 3 times $\frac{2}{3}$ of what number?
5. $\frac{5}{8}$ of 48 are 3 times $\frac{2}{5}$ of what number?
6. $\frac{3}{4}$ of 40 are $\frac{2}{3}$ of 5 times what number?

Analysis.—First, $\frac{1}{4}$ of 40 is 10, and $\frac{3}{4}$ are 3 times 10 or 30. Now, if 30 is $\frac{2}{3}$ of a number, $\frac{1}{3}$ is $\frac{1}{2}$ of 30, which is 15, and $\frac{3}{3}$ are 3 times 15, or 45. Finally, 45 is 5 times $\frac{1}{5}$ of 45, which is 9. Therefore, $\frac{3}{4}$ of 40 are $\frac{2}{3}$ of 5 times 9.

7. A man paid $50 for a sleigh, and $\frac{3}{5}$ of the cost of his sleigh was $\frac{2}{3}$ of 3 times the cost of his harness: what was the cost of his harness?

8. A paid $\frac{7}{12}$ of $96 for a horse, which was 3 times $\frac{4}{5}$ the cost of his buggy: what was the price of his buggy?

9. A man invested $1000 in business; $\frac{4}{5}$ of his stock was $\frac{3}{4}$ of 10 times his profit: what was his profit?

68. Ex. 1. $\frac{3}{4}$ of 32 are $\frac{6}{8}$ of how many fifths of 40?

Analysis.—First, $\frac{1}{4}$ of 32 is 8, and $\frac{3}{4}$ are 3 times 8 or 24 Next, since 24 is $\frac{6}{8}$ of a certain number, $\frac{1}{8}$ must be $\frac{1}{6}$ of 24, which is 4, and $\frac{8}{8}$ are 8 times 4 or 32. Finally, $\frac{1}{5}$ of 40 is 8, and 8 is contained in 32, 4 times. Therefore, $\frac{3}{4}$ of 32 are $\frac{6}{8}$ of 4 fifths of 40.

2. $\frac{4}{5}$ of 30 are $\frac{6}{7}$ of how many fifths of 45?
3. $\frac{3}{7}$ of 42 are $\frac{3}{10}$ of how many sixths of 48?
4. $\frac{5}{6}$ of 54 are $\frac{9}{11}$ of how many tenths of 80?
5. $\frac{2}{3}$ of 27 are $\frac{6}{8}$ of how many fourths of 32?

QUESTIONS FOR REVIEW.

69. Ex. 1. A certain tree casts a shadow 48 feet long, which is $\frac{4}{5}$ of its height: what is the height of the tree?

2. A traveler inquiring how far it was from A to C, was answered, that from A to B it is 63 miles, which are $\frac{9}{10}$ of the distance from B to C: how far is it from B to C? How far from A to C?

3. A lad had 45 marbles, which was $\frac{5}{7}$ of the numbers of his companion: how many had his companion? How many had both?

4. Eight feet of a liberty pole are in the ground, which are $\frac{2}{12}$ of its whole length: what is its length?

5. Charles being asked how many blackberries he had, answered that his companion had $5\frac{3}{4}$ quarts, which were $\frac{3}{4}$ as many as he had: how many quarts had he?

6. A well-digger, after going down 15 feet, found this was but $\frac{3}{5}$ of the distance from the surface to the point where he would find water: how long did it take him to dig the well, allowing he dug 6 feet per day?

7. If the silk for a lady's dress costs $31, and $\frac{4}{5}$ of the cost of the dress is twice the cost of the trimmings, what is the cost of the trimmings?

8. A man paid $600 rent, which was $\frac{2}{3}$ of 4 times $\frac{1}{2}$ of his other expenses: what were his other expenses?

9. A goldsmith sold a watch for $60, which was $\frac{7}{4}$ of $\frac{4}{5}$ of what it cost him: what did the watch cost him?

10. A lad gave 50 cents for an Astronomy, which were $\frac{2}{3}$ of 5 times all the money he had: how much had he?

11. Henry has 45 marbles, and $\frac{4}{5}$ of his marbles equal $\frac{2}{3}$ of $\frac{3}{4}$ of George's marbles: how many marbles has George?

12. Two lads talking of their ages, one said he was 15 years old; the other said that 4 times $\frac{2}{3}$ of that was $2\frac{1}{2}$ times his age: what was his age?

13. A market-man said that in one basket were 5 dozen eggs, and $\frac{3}{4}$ of this number equal $\frac{5}{8}$ of the number in the other: how many eggs had he in the other basket? How many had he in both?

14. John sold 25 apples, which was $\frac{5}{8}$ of what he now has, and said if any one would tell him how many remained, he would divide them equally among the company, which consisted of 5 persons: how many apples has he? How many would each receive?

15. A school-girl being asked how much she paid for her slate and Arithmetic, answered that she gave 15 cents for her slate, and that $\frac{4}{5}$ the cost of the former was $\frac{2}{7}$ the cost of the latter: what did she pay for her Arithmetic?

16. A's age, which is 40 years, is $\frac{4}{5}$ of $\frac{5}{8}$ of B's age: how old is B?

17. A can do a piece of work in 63 days, and $\frac{2}{7}$ of this is equal to $\frac{3}{4}$ of the time it will take B and C together to do it: how long will it take B and C to do it?

18. A general after losing $\frac{1}{3}$ of his regiment in battle, and $\frac{1}{4}$ by sickness, had 1000 men left: how many soldiers had he at first?

CHAPTER VIII.

DECIMALS AND UNITED STATES MONEY.

69. Preliminary Exercises in Decimals.

Ex. 1. If a sheet of paper is divided into 10 equal parts, what part of a sheet is 1 of them?

Ans. One *tenth*, or $\frac{1}{10}$ of a sheet.

2. If one of these *tenths* is divided into 10 other equal parts, what part of a sheet is one of them?

Ans. One *hundredth*; for, $\frac{1}{10}$ of $\frac{1}{10} = \frac{1}{100}$ of a sheet.

3. If one of these *hundredths* is divided into 10 other equal parts, what part of a sheet is one of them?

Ans. One *thousandth*; for, $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10} = \frac{1}{1000}$.

4. What is meant by a tenth? By a hundredth? A thousandth?

5. When a unit is divided into *tenths, hundredths, thousandths*, etc., what are these fractions called?

Ans. ***Decimal Fractions.***

6. How are decimal fractions commonly expressed?

Ans. By *writing the numerator only*, with a *decimal point* on the left; as, .1, one tenth; .01, one hundredth; .001, one thousandth, etc.

7. What is the denominator of a decimal fraction?

Ans. It is 1 with as *many ciphers annexed* as there are figures in the numerator.

8. How do the orders of whole numbers *increase*?

Ans. From *right to left by the scale* of *ten*; that is, 10 of a *lower* order make 1 of the next *higher*.

9. How do the orders *below* units *decrease*?

Ans. From *left* to *right* by the *scale* of *ten;* that is, each succeeding order is 1 tenth of the preceding order.

10. How many tenths make a unit? How many hundredths, a tenth? Thousandths, a hundredth?

70. Reducing Common Fractions to Decimals.

Ex. 1. To how many tenths are $\frac{3}{5}$ equal?

Analysis.—1 equals 10 tenths, hence $\frac{1}{5}$ of 1 equals $\frac{1}{5}$ of 10 tenths, or 2 tenths, and $\frac{3}{5}$ are 3 times 2 tenths, or 6 tenths, *Ans.*

NOTE.—This operation is the same in principle as reducing a fraction to any required denominator. (Art. 37.)

2. To how many tenths are $\frac{2}{5}$ equal? $\frac{4}{5}$?

3. To how many tenths are $\frac{5}{5}$ equal? $\frac{7}{5}$? $\frac{8}{5}$?

4. To how many hundredths are $\frac{3}{5}$ equal?

Analysis.—1 is equal to 100 hundredths; hence 1 fifth of 1 equals 1 fifth of 100 hundredths, which is 20 hundredths, and 3 fifths are 3 times 20, or 60 hundredths, *Ans.*

5. Reduce $\frac{1}{4}$ to hundredths. Reduce $\frac{3}{4}$ to hundredths.

6. Reduce $\frac{9}{30}$ to tenths. Reduce $\frac{4}{5}$ to hundredths.

7. Reduce $\frac{5}{25}$ to tenths; also to hundredths.

8. Reduce $\frac{8}{40}$ to tenths; also to hundredths.

9. Which is the greater, $\frac{2}{5}\frac{1}{5}$ or 60 hundredths?

71. Reducing Decimals to Common Fractions.

Ex. 1. Reduce 8 tenths to a com. frac. in lowest terms

Analysis.—The decimal, 8 tenths, expressed as a common fraction, is $\frac{8}{10}$. Reduced to the lowest terms, $\frac{8}{10} = \frac{4}{5}$, *Ans.*

Reduce the following to com. fractions in lowest terms:

2. Reduce 5 tenths.

3. Reduce 20 hundredths.

4. Reduce 25 hundredths.

5. Reduce 35 hundredths.

6. Reduce 40 hundredths.

7. Reduce 80 thousandths.

UNITED STATES MONEY.

72. ***United States Money*** is the national currency of the United States. It is also called *Federal Money.*

TABLE.

10 mills (*m.*)	make	1 cent, *ct.*
10 cents	"	1 dime, *d.*
10 d., or 100 cts.	"	1 dollar, $, or *dol.*
10 dollars	"	1 eagle, *E.*

NOTE.—The *denominations* of U. S. money, like the *orders* of numbers, *increase* and *decrease* by the *scale* of 10. It is thence called *Decimal Currency.*

2. How is U. S. Money written?

Ans. ***Dollars*** are written as *whole numbers*, with the sign $ prefixed to them.

Cents are written in the *first two* places on the right of the *decimal point;* as *hundredths* of a dollar.

Thus, 13 dollars 25 cents are written $13.25.

Mills are written in the *third* place on the right; as *thousandths* of a dollar.

Thus, 25 dollars 3 cents 8 mills are written $25.038.

73. Reducing Dollars to Cents and Mills.

Ex. 1. How many cents are there in $3?

Analysis.—Since in $1 there are 100 cents, in $3 there are 3 times 100 or 300 cents, *Ans.*

2. How many cents in 5 dollars? 7 dollars? 6 dollars?

3. How many cents in 8 dollars? In 10 dollars? In 15 dollars?

74. Reducing Cents and Mills to Dollars.

4. How many mills in $2? In $4? In $6? In $9?

5. How many dollars in 350 cents?

Analysis.—Since 100 cents make $1, 350 cents are as many dollars as 100 is contained times in 350, or $3\frac{1}{2}$. *Ans.*

6. How many dollars in 500 cts.? In 700 cts.? In 1500 cts.?

7. How many dollars in 425 cts.? In 620 cts.? In 712$\frac{1}{2}$ cts.?

8. How many dollars in 875 cents? In 937$\frac{1}{2}$ cents? In 1200 cents?

9. How many dollars in 1200 cents? In 2500 cents? In 2700 cents?

10. How many dollars in 1000 mills? In 2000 mills? In 5000 mills?

75. Finding Aliquot parts of a dollar.*

Ex. 1. How many cents in 1 half dollar?

Analysis.—Since there are 100 cents in $1, in 1 half dollar there must be ½ of 100 cents, or 50 cents. *Ans.*

2. How many cents in 1 third of a dollar?

3. How many cents in $\$\frac{1}{4}$? In $\$\frac{1}{3}$?

4. How many cents in $\$\frac{1}{6}$? In $\$\frac{1}{8}$?

5. How many cents in $\$\frac{1}{10}$? In $\$\frac{1}{25}$?

6. What part of a dollar is 50 cents?

Analysis.—50 cents are $\$\frac{50}{100}$, and $\$\frac{50}{100} = \$\frac{1}{2}$, *Ans.*

7. What part of a dollar is 25 cents? 20 cts? 10 cts.? 5 cts.? 4 cts.? 2 cts.?

8. What part of a dollar is 33⅓ cents? 16⅔ cts.?

9. What part of a dollar is 12½ cts.? 6¼ cts.?

10. What part of a dollar is 8⅓ cts.? 37½ cts.? 62½ cts.? 75 cts.? 87½ cts.?

BUSINESS OPERATIONS.

76. Ex. 1. A lady with $25 bought a hat for $14 and a pair of shoes for $8: how much money had she left?

2. A boy earned 50 cts. by doing errands, and wished to buy toys for his sisters. He paid 7 cts. for a rattle-box, and 15 cts. for a doll: how much money had he left?

Analysis.—15 cts. and 7 cts. make 22 cts. Now 22 cts. are 3 less than 25 cts., and 25 cts. from 50 cts. leave 25 cts., and 3 more make 28 cents: therefore he had 28 cts. left.

3. If you pay 25 cts. for a book, 20 cts. for a pencil case, and 30 cts. for an inkstand, how much change would you get from a dollar bill?

* An *Aliquot part* of a number is an *exact divisor* of that number.

4. A little girl was sent to a thread and needle store to buy trimmings for a cloak, and had $1\frac{1}{2}$ in her purse. She bought 12 yds. braid at 4 cents a yd., 6 yds. cambric at 10 cts., and $1\frac{1}{2}$ doz. buttons at 24 cts. a dozen: how much change should she bring back?

5. A young man bought a suit of clothes giving $15 for the coat, $8 for pants, and $3 for a vest; he had three 10 dollar bills, and after paying for the clothes took the balance in handkerchiefs at 50 cents apiece: how many did he get?

6. A young lady having $5\frac{1}{2}$, bought a pair of gloves for $2, 3 yds. of lace at $33\frac{1}{3}$ cts. a yd., and a thimble for 50 cts.: how much money had she left?

7. A man wishing to make Christmas presents to his family, bought for his wife a writing-desk at $18, a pair of bracelets for his daughter at $15, and a pair of sleeve-buttons for his son at $12. If he paid for them with a 50-dollar bill, how much change should he receive?

8. If I buy $6\frac{1}{2}$ yards of silk at $1.50 a yard, 3 yards of velvet at $4 a yard, and give a check for $25, how much change should I have?

9. A school-girl bought a Geography for $1\frac{1}{4}$, a History for $1.10, an Algebra for $1.12\frac{1}{2}$, and gave a 5-dollar bill: how much change did she have?

10. A mother having $35 to lay out in summer dresses, wished to buy an equal quantity of printed cambric at $37\frac{1}{2}$ cts. a yard, and grenadine at $1\frac{1}{4}$ a yard: how many whole yards of each could she buy, and how much change would she have?

11. If you have $50 and wish to get a shawl for $15, and a silk dress at $1.50 a yard., how many yards can you buy?

SHORT METHODS.

77. Finding the *Cost* of a given quantity, when the price of *one* is an Aliquot part of $1.

NOTE.—The *Aliquot* parts of a dollar may be tabulated in the following manner, and if fixed in the mind of the pupil, will be a valuable help in practical life:

50 cts. = $\frac{1}{2}$ of $1.	33$\frac{1}{3}$ cts. = $\frac{1}{3}$ of $1.
25 cts. = $\frac{1}{4}$ of $1.	16$\frac{2}{3}$ cts. = $\frac{1}{6}$ of $1.
20 cts. = $\frac{1}{5}$ of $1.	12$\frac{1}{2}$ cts. = $\frac{1}{8}$ of $1.
10 cts. = $\frac{1}{10}$ of $1.	8$\frac{1}{3}$ cts. = $\frac{1}{12}$ of $1.
5 cts. = $\frac{1}{20}$ of $1.	6$\frac{1}{4}$ cts. = $\frac{1}{16}$ of $1.

EX. 1. At 25 cents a yard, what will 9 yards of muslin cost?

Analysis.—25 cents are $\frac{1}{4}$. At $1 a yard 9 yards will cost $9; and at $\frac{1}{4}$ a yard, 9 yards will cost $\frac{1}{4}$ of $9 = 2\frac{1}{4}$ or $2.25, *Ans.*

2. What cost 8 bushels of apples, at $.50 a bushel?

3. What cost 25 pounds of honey, at $.20 a pound?

4. At $.10 apiece, what will 50 pies come to?

5. At $.12$\frac{1}{2}$ apiece, what must I pay for 2 dozen slates?

6. A hatter sold 4 dozen caps, at $.33$\frac{1}{3}$ apiece: what was the amount of his bill?

7. A merchant sold 60 yards of calico, at 16$\frac{2}{3}$ cents a yard: how much did it come to?

8. An apple-woman sold 100 oranges, at 6$\frac{1}{4}$ cents apiece: how much did she receive for them?

9. A provision dealer bought 500 pounds of cheese, at $.10 a pound: what was the amount of his bill?

10. If railroad fare is $.05 a mile, what will be the fare for 1000 miles?

11. If delaine is $.50 a yard, what must I pay for 150 yards?

12. A market-man sold 10 dozen chickens, at 33$\frac{1}{3}$ cents apiece: what did they come to?

13. A dairy-woman sold 175 pounds of butter, at 25 cts. a pound: how much did she receive for it?

14. How much will 50 pounds of maple sugar come to, at 16⅔ cents a pound?

15. If a butcher pays $.75 apiece for 100 turkeys, how much will all cost him?

78. Finding the *Quantity*, when the cost is given and the price of *one* is an Aliquot part of $1.

Ex. 1. How many writing-books, at 12½ cents each, can be bought for $6?

Analysis.—12½ cents are ⅛ of a dollar. Now, if $⅛ will buy 1 writing-book, $6 will buy as many as there are eighths in 6, and 8 times 6 are 48. Therefore, $6 will buy 48 writing-books.

2. How many bushels of apples, at $.50 a bushel, can be had for $30?

3. How many Spellers, at $.25 will $15 buy?

4. A grocer laid out $12 for pepper which was $.20 a pound: how many pounds did he get?

5. A pedlar received $10 for pocket-combs which were 12½ cents apiece: how many did he sell?

6. A housekeeper laid out $30 for butter which was worth 33⅓ cents a pound: how many pounds did she buy?

7. Henry invested $4 in lead pencils which were 6¼ cts. apiece: how many did he buy?

8. A lady spent $12 for muslin which was 16⅔ cents a yard: how many yards did she buy?

9. A market-man sold a lot of chickens, at 33⅓ cents each, and received $12 for them: how many did he sell?

10. How many times are 16⅔ cents contained in $11?

11. A company of boys spent $7 for their lunch, which was 20 cents apiece: of how many did the company consist?

12. How many times are 33⅓ cents contained in $25?

79. To multiply a Mixed Number ending with $\frac{1}{2}$, by itself.

Ex. 1. What cost $2\frac{1}{2}$ yards of lace, at $\$2\frac{1}{2}$ a yard?

Analysis.—First $2\frac{1}{2}$ multiplied by $2 = 5$, and $2\frac{1}{2} \times \frac{1}{2} = 1\frac{1}{4}$. Now $5 + 1\frac{1}{4} = 6\frac{1}{4}$, *Ans.*

NOTE.—The operation is simplified by multiplying the integral part by 1 more than itself and annexing $\frac{1}{4}$ to the result.

Thus: $2 \times 3\ (2+1) = 6$, and $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. Now $6 + \frac{1}{4} = 6\frac{1}{4}$, *Ans.*

2. Multiply $5\frac{1}{2}$ by $5\frac{1}{2}$.

3. Multiply $6\frac{1}{2}$ by $6\frac{1}{2}$.

4. What cost $8\frac{1}{2}$ yards of muslin at $8\frac{1}{2}$ cents a yard?

5. At $11\frac{1}{2}$ cents a yard, what will $11\frac{1}{2}$ yards of calico cost?

6. At $\$9\frac{1}{2}$ a yard what will $9\frac{1}{2}$ yards of silk velvet cost?

80. To multiply any two Mixed Numbers together, whose fractional part is $\frac{1}{2}$.

Ex. 1. What is the product of $4\frac{1}{2}$ into $2\frac{1}{2}$?

Analysis.—$4\frac{1}{2} \times 2 = 9$, and $4\frac{1}{2} \times \frac{1}{2} = 2\frac{1}{4}$, which added to 9 make $11\frac{1}{4}$, the answer required.

NOTE.—1. The operation is simplified if we multiply the integral parts together, to the product add half their sum, and annex $\frac{1}{4}$ to the result.

Thus, the product of $4 \times 2 = 8$; half the sum of $4 + 2$, or 6 is 3, which added to 8 makes 11, and $11 + \frac{1}{4} = 11\frac{1}{4}$, *Ans.*

2. What is the product of $6\frac{1}{2}$ into $2\frac{1}{2}$?

3. At $6\frac{1}{2}$ cents a yard, what cost $4\frac{1}{2}$ yards of braid?

4. What cost $6\frac{1}{2}$ bushels of apples, at $8\frac{1}{2}$ shillings?

NOTE.—2. If the *sum* of the integral parts is an *odd* number, to their product add *half* of the next *smaller* number, and to this result annex $\frac{1}{4}$.

5. If you multiply $5\frac{1}{2}$ by $4\frac{1}{2}$, what is the product?

SOLUTION.—The sum of $5 + 4$ or 9, is an odd number, and the next smaller number is 8, half of which is 4; therefore, to the product of 5×4 or 20, we add 4, which makes 24, and $24 + \frac{1}{4} = 24\frac{1}{4}$, *Ans.*

6. What is the product of $10\frac{1}{2}$ into $3\frac{1}{2}$?

7. What is the product of $3\frac{1}{2}$ into $6\frac{1}{2}$?

8. What cost $5\frac{1}{2}$ oranges at $3\frac{1}{2}$ cents apiece?

9. If you multiply $10\frac{1}{2}$ by $3\frac{1}{2}$, what is the product?

10. If a man spends $12\frac{1}{2}$ cents a day for cigars, how much will he spend in $4\frac{1}{2}$ days?

11. What is the product of $11\frac{1}{2}$ into $3\frac{1}{2}$?

12. What is the product of $20\frac{1}{2}$ into $4\frac{1}{2}$?

13. At $30\frac{1}{2}$ cts., what cost $5\frac{1}{2}$ gallons of milk?

14. What is the worth of $50\frac{1}{2}$ cords of wood at $10\frac{1}{2}$ shillings per cord?

TRAFFIC, OR EXCHANGE OF COMMODITIES.

81. Ex. 1. A merchant sold 8 pounds of tea at 6 shillings a pound, and took his pay in oats at 4 shillings a bushel: how many oats did he receive?

Analysis.—8 pounds of tea are worth 8 times as much as 1 pound, and 8 times 6s. are 48s. Now, if 4s. will pay for 1 bu. of oats, 48s. will pay for as many bushels as 4s. are contained times in 48s., or 12 bushels, *Ans.*

NOTE.—In this and similar questions the rule is this:

Find what the *goods*, whose quality and price are given, *amount* to; then find *what quantity* of the article taken in exchange, is *equivalent* to this amount.

2. How many pears, at 4 cents each must you give for 6 oranges, at 6 cents apiece?

3. How many sheep, at $3 apiece will it take to pay for 2 tons of hay, at $15 a ton?

4. A farmer bought 8 pair of boots, at $6 a pair, and agreed to pay for them in wood, at $4 a cord: how many cords did it take?

5. Bought 6 barrels of flour, at $10, and paid for it in coal, at $5 a ton: how many tons of coal did it take to pay for the flour?

6. A farmer bought 6 yards of muslin, at 12 cents, and paid for it in cheese, at 9 cents a pound: how much cheese paid for it?

7. How many cows, at $20 each, will it take to pay for 10 tons of hay, at $12 a ton?

8. How many pineapples, at 12 cents can be bought with 8 quarts of blackberries, at 9 cents a quart?

9. How many dimes will it take to pay for 6 melons, at 15 cents each?

10. How many colts, at $50 each, will it take to pay for 4 cows, at $25?

11. If I buy 8 barrels of flour, at $6, and sell it at $10, how much shall I make by the operation?

Analysis.—Since I buy at $6 and sell at $10, I make $4 on a barrel. Now if I make $4 on 1 barrel, on 8 barrels I shall make 8 times $4, which are $32. Therefore, etc.

12. A drover bought 10 lambs, at 12s. a head, and sold them, at 13s.: how much did he make?

13. A market-woman buys celery, at 8 cts. a bunch, and sells it at 15 cts.: what does she make on 12 bunches?

14. A druggist buys fancy soap at 9 cts. a cake, and sells it at 15 cts.: what does he make on 2 dozen cakes?

15. If 12 tons of coal cost $108, how many bags of coffee, at $6 a bag, would 6 tons buy?

16. Henry bought 4 pumpkins, at 25 cents apiece, and sold them at 20 cents: how much did he lose?

17. George bought 12 oranges at 6 cents apiece, and exchanged them for 4 quarts of peanuts, which he sold at 20 cents a quart: how much did he make or lose?

18. Bought 15 tons of hay at $15, and exchanged 5 tons of it for 3 cows: what did each cow cost?

19. Bought 9 barrels of flour for $72, and gave 6 of them for 12 yards of cloth: what was the cloth a yard?

20. Bought 8 saddles for $96, and gave 6 of them for 12 pairs of boots: what was the cost of a pair of boots?

CHAPTER IX.

COMPOUND OR DENOMINATE NUMBERS.

82. English, or Sterling Money.*

4 farthings (*qr.* or *far.*)	are	1 penny,	*d.*
12 pence	"	1 shilling,	*s.*
20 shillings	"	1 pound,	£.
21 shillings	"	1 guinea,	*g.*

NOTES.—1. The *pound sterling* is represented by a gold coin called a *sovereign*, and its value is $4.86⅔.

2. Farthings are commonly written as fractions of a penny. Thus, 6d. 1 far. are written 6¼d.

Sovereign.

Ex. 1. How many shillings in £½? In £¼? £$\frac{1}{10}$?

2. What part of a shilling is 6 pence? 4d.? 3d.? 2d.?

3. What part of a penny is 2 far.? 3 far.?

83. Troy Weight.

24 grains (*gr.*)	are	1 pennyweight,	*pwt.*
20 pennyweights	"	1 ounce,	*oz.*
12 ounces	"	1 pound,	*lb.*

lb. oz. pwt. gr.

4. What part of a pound Troy is 6 oz.? 4 oz.? 3 oz.? 2 oz.?

* For United States or Federal Money, see page 82.

5. What part of an ounce is 10 pwts.? 5 pwts.? 4 pwt.? 2 pwts.?

6. What part of a pennyweight is 12 grains? 8 grs.? 6 grs.? 4 grs.? 3 grs.? 2 grs.?

84. Avoirdupois Weight.

16 ounces (*oz.*)	are	1 pound,	*lb.*
100 pounds	"	1 hundredweight,	*cwt.*
20 cwt., or 2000 lbs.,	"	1 ton,	*T.*

7. What part of a ton is 10 hundredweight? 5 cwt.? 4 cwt.? 2 cwt.?

8. How many ounces in $\frac{1}{2}$ pound Avoirdupois? In $\frac{1}{4}$ lb.? In $\frac{1}{8}$ lb.?

9. What part of a hundredweight is 50 pounds? 25 lbs.? $33\frac{1}{3}$ lbs.?

85. Long Measure.

12 inches (*in.*)	are	1 foot,	*ft.*
3 feet	"	1 yard,	*yd.*
$16\frac{1}{2}$ feet, or $5\frac{1}{2}$ yards	"	1 rod, perch, or pole,	*r.* or *p.*
40 rods	"	1 furlong,	*fur.*
8 fur., or 320 rods	"	1 mile,	*m.*
3 miles	"	1 league,	*l.*

NOTE.—The inch is commonly divided into *halves, fourths, eighths,* or *tenths;* sometimes into *twelfths,* called *lines.*

10. What part of a foot is 6 inches? 4 in.? 3 in.? 2 in.?

11. What part of a furlong is 20 rods? 10 rd.? 8 rd.? 5 rd.?

86. Cloth Measure.

3 feet, or 36 in.	are	1 yard,	*yd.*
18 inches	"	1 half yard,	$\frac{1}{2}$ *yd.*
9 inches	"	1 quarter yard,	$\frac{1}{4}$ *yd.*
$4\frac{1}{2}$ inches	"	1 eighth yard,	$\frac{1}{8}$ *yd.*
$2\frac{1}{4}$ inches	"	1 sixteenth yd.,	$\frac{1}{16}$ *yd.*

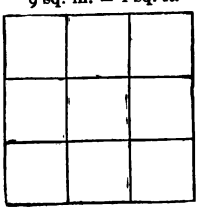
12. What part of a yard is 2 feet? 1 ft.? $1\frac{1}{2}$ ft.?

13. What part of a yard is 18 inches? 12 in.? 9 in.? 6 in.? 4 in.? 3 in.?

87. Square Measure.

144 square in. (*sq. in.*)	are	1 square foot,	*sq. ft.*
9 square feet	"	1 square yard,	*sq. yd.*
$30\frac{1}{4}$ square yards, or $272\frac{1}{4}$ square feet	"	1 square rod, perch, or pole,	*sq. r.*
160 square rods	"	1 acre,	*A.*
640 acres	"	1 square mile,	*sq. m.*

NOTES.—1. A ***square*** is a rectilinear figure which has *four* equal sides, and *four right angles.*



2. A ***square inch*** is a square, each side of which is *one* inch in length.

3. A ***square yard*** is a square, each side of which is *one* yard in length.

4. The *corners* of any square figure, also of a book, a table, a room, etc., are *right angles.*

5. The *area* of rectangular surfaces is found by multiplying the *length* and *breadth* together.

14. What part of a square foot is 72 square inches? 48 sq. in.? 36 sq. in.? 24 sq. in.? 12 sq. in.? 6 sq. in.?

15. What part of a square yard is 3 square feet? 6 sq. ft.? 1 sq. ft.?

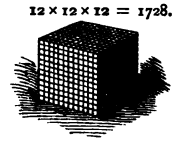
16. How many square rods in $\frac{1}{2}$ acre? In $\frac{1}{4}$ A.? In $\frac{1}{8}$ A.? In $\frac{1}{10}$ A.?

88. Cubic Measure.

1728 cubic inches (*cu. in.*)	are	1 cubic foot,	*cu. ft.*
27 cubic feet	"	1 cubic yard,	*cu. yd.*
128 cubic feet	"	1 cord,	*C.*

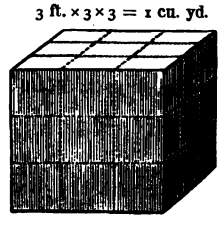
NOTES.—1. A ***cube*** is a regular solid bounded by *six equal squares*, called its faces.

2. A ***cubic inch*** is a cube, each side of which is a square inch.



3. A ***cubic foot*** is a cube, each side of which is a square foot.

4. A ***cubic yard*** is a cube, each side of which is a square yard.



5. A ***Cord*** of wood is a pile 8 ft. long, 4 ft. wide, and 4 ft. high; for, $8 \times 4 \times 4 = 128$.

6. A *Cord* Foot = 16 cu. ft., and is 1 ft. long, 4 ft. wide, and 4 ft. high. 8 cord feet make 1 cord.

7. The *contents* of a *rectangular solid* are found, by *multiplying the length, breadth,* and *thickness* together.

17. What part of a cubic foot is 144 cubic inches? 12 cu. in.?

18. What part of a cubic yard is 9 cubic feet? 3 cu. ft.? 1 cu. ft.?

19. What part of a cord is 64 cu. ft.? 4 cu. ft.? 16 cu. ft.?

89. Dry Measure.

2 pints (*pt.*)	are	1 quart,	*qt.*
8 quarts	"	1 peck,	*pt.*
4 pecks, or 32 qts.,	"	1 bushel,	*bu.*
36 bushels	"	1 chaldron,	*ch.*

NOTE.—The *dry* quart is equal to $1\frac{1}{8}$ liquid quart nearly.

20. What part of a bushel is 16 quarts? 8 qts.? 4 qts.? 2 qts.?

21. What part of a bushel is 1 peck? 2 pks.? 3 pks.?

90. Liquid Measure.

4 gills (*gi.*)	are	1 pint,	*pt.*
2 pints, or 8 gills,	"	1 quart,	*qt.*
4 quarts	"	1 gallon,	*gal.*
$31\frac{1}{2}$ gallons	"	1 barrel,	*bar.*, or *bbl.*
63 gallons, or 2 bbls.,	"	1 hogshead,	*hhd.*

NOTE.—Liquid Measure is often called *Wine Measure*.

22. What part of a gallon is 2 quarts? 3 qts.?

23. What part of a quart is 4 gills? 2 gills?

91. Time.

60 seconds (*sec.*)	are	1 minute,	*min.*
60 minutes	"	1 hour,	*h.*
24 hours	"	1 day,	*d.*
7 days	"	1 week,	*w.*
365 days, or 52 w. and 1 d.,	"	1 common year,	*c. y.*
366 days	"	1 leap year,	*l. y.*
12 calendar months (*mo.*)	"	1 civil year,	*y.*
100 years	"	1 century,	*cen.*

NOTE.—In most business transactions, 30 days are considered a *month*. Four weeks are sometimes called a *lunar month*.

24. What part of a day is 12 hours? 8 hrs.? 6 hrs.? 4 hrs.? 3 hrs.? 2 hrs.?

25. What part of a business month is 15 days? 10 d.? 6 d.? 5 d.? 3 d.?

26. What part of year is 6 months? 4 m.? 3 m.? 2 m.?

27. What part of a century is 50 years? 25 y.? 20 y.? 10 y.? 5 y.? 4 y.? 2 y.?

28. How many minutes in $\frac{1}{2}$ hour? In $\frac{1}{3}$ hr.? $\frac{1}{4}$ hr.? $\frac{1}{5}$ hr.? $\frac{1}{6}$ hr.? $\frac{1}{10}$ hr.? $\frac{1}{12}$ hr.?

92. Miscellaneous Tables.

12 things are 1 dozen.	12 gross are 1 great gross.
12 dozen " 1 gross.	20 things " 1 score.
24 sheets are 1 quire of paper.	2 reams are 1 bundle.
20 quires " 1 ream.	5 bundles " 1 bale
2 leaves are 1 folio.	12 leaves are 1 duodecimo or 12mo.
4 leaves " 1 quarto or 4to.	18 leaves " 1 eighteen mo.
8 leaves " 1 octavo or 8vo.	24 leaves " 1 twenty-four mo.

NOTE.—The terms *folio, quarto, octavo*, etc., denote the *number* of leaves into which a sheet of paper is folded in making books.

REDUCTION.

93. Reducing Higher denominations to Lower.

Ex. 1. In £3 12s., how many shillings?

Analysis.—Since there are 20 shillings in a pound, there must be 20 times as many shillings as pounds, plus the given shillings. Now 20 times 3 are 60s., and 12s. make 72s.

Or, thus: In £1 there are 20s., and in £3 there are 3 times 20s., or 60s., and 12s. make 72s., *Ans.*

2. How many farthings in 9d.? In 12d.? 15d.?
3. In 8s., how many pence? In 7s.? 10s.? 12s.? 30s.?
4. In £5, how many shillings? In £6? £15? £50?
5. In £3 and 5 shillings, how many shillings?
6. In 8 feet, how many inches? In 6½ ft.? In 10½ ft.?
7. How many yards in 8 rods? In 9 r.? 12 r.?
8. How many feet in 3 rods? In 4 r.? 5 r.?

94. Reducing Lower denominations to Higher.

9. How many pence in 43 farthings?

Analysis.—In 4 far. there is 1 penny, hence in 43 far. there are as many pence as 4 is contained times in 43, or 10¾ far., *Ans.*

10. How many shillings in 41d.? In 64d.? 87d.?
11. How many pounds in 68s.? In 83s.? 110s.?
12. In 64 inches, how many feet? In 75 in.? 112 in.?
13. In 28 feet, how many yards? In 33 ft.? 50 ft.?
14. In 15 yards, how many rods? In 17 yds.? 50 yds.?

95. Promiscuous Exercises in Reduction.

Ex. 1. In 4 lbs. Troy, how many oz.?
2. In 5 pwts., how many grains?
3. In 6 oz., how many pennyweights?
4. How many pounds in 65 oz. Troy?
5. How many ounces in 84 pwts.?

6. How many oz. in 3 lbs. Avoidupois? In 4 pounds?
7. How many oz. in 5 pounds 3 ounces?
8. How many oz. in 10 pounds 8 ounces?
9. In 42 hundredweight, how many tens? In 65 cwt.?
10. In 57 ounces, how many pounds? In 85 ounces?
11. How many cwt. in 3 tons? In 4 tons? In 6 tons?
12. How many feet in 72 inches? How many yards?
13. How many inches in 1 yard and a half?
14. How many inches in 2 yards and 1 quarter?
15. What part of a yard is 27 inches?
16. How many yards in 17 eighths? In 35 eighths?
17. How many inches in 2 square feet? In 10 sq. ft.?
18. How many sq. yards in 45 sq. feet? In 84 sq. ft.?
19. How many sq. yds. in 2 sq. rods? In 5 sq. rods?
20. In 3 acres, how many rods? In 4 acres? 10 acres?
21. In 320 sq. rods, how many acres? In 480 sq. rods?
22. How many cubic yards in 54 cubic feet?
23. How many cords in 96 cord feet?
24. How many cord feet in 5 cords?
25. In 26 pints, how many quarts? In 42 pints?
26. In 32 quarts, how many pecks? In 46 quarts?
27. In 27 pecks, how many bushels? In 33 pecks?
28. In 4 bushels, how many pecks? How many quarts?
29. In 6 pecks and 2 quarts, how many pints?
30. How many gills in 12 pints? In 16 pints?
31. How many gallons in 2 barrels? In 5 barrels?
32. In 2 hogsheads, how many galls.? In 10 hhds.?
33. In 27 pecks, how many bushels? In 33 pecks?
34. In 4 bushels, how many quarts? How many pints?
35. How many days in 7 weeks? In 9 w.? 11 w.? 20 w.?
36. In 1 hour, how many seconds? In 2 hours?
37. In 3 days, how many hours? In 5 days? 10 days?
38. In 360 seconds, how many minutes?
39. How many hours in 720 minutes?
40. How many calendar months in 5 yrs. and 7 mos.?

96. Fractional Compound Numbers.

Ex. 1. In £$\frac{2}{3}$, how many shillings and pence?

Analysis.—In £1 there are 20s., and in £$\frac{2}{3}$ there are $\frac{2}{3}$ of 20s. Now $\frac{1}{3}$ of 20s. is 6$\frac{2}{3}$s., and 2 thirds are twice as many, or 13$\frac{1}{3}$s.

Again, in 1s. there are 12d., and in $\frac{1}{3}$s. there is $\frac{1}{3}$ of 12d., which is 4d. Therefore, in £$\frac{2}{3}$ there are 13s. and 4d.

2. In $\frac{3}{4}$ of a shilling, how many farthings?
3. How many farthings in £$\frac{1}{4}$? In £$\frac{3}{4}$? In £$\frac{2}{3}$? £$\frac{3}{5}$?
4. How many pence in £$\frac{1}{8}$? £$\frac{3}{8}$? £$\frac{3}{10}$? £$\frac{7}{10}$? $\frac{5}{12}$? $\frac{3}{15}$?
5. How many hundredweight in $\frac{3}{4}$ ton? In $\frac{4}{5}$ ton?
6. How many pounds in $\frac{1}{4}$ ton? $\frac{3}{4}$ ton? $\frac{7}{10}$ ton?
7. How many quarts in $\frac{3}{4}$ bushel? In $\frac{5}{8}$ bushel?
8. How many rods in $\frac{1}{2}$ acre? In $\frac{1}{4}$ A.? $\frac{3}{4}$ A.? $\frac{3}{8}$ A.?
9. What part of £1 is 6 shillings and 8 pence?

Analysis.—6s. 8d. = 80d., and £1 = 240d. Now 80d. are $\frac{80}{240}$, or $\frac{1}{3}$ of 240d. Therefore, 6s. 8d. are $\frac{1}{3}$ of £1.

10. What part of £1 is 4s. and 8d.
11. What part of £1 is 5 shillings? Is 4 shillings?
12. What part of £1 is 3 shillings and 4 pence?
13. What part of £1 is 2 shillings and 6 pence?
14. What part of £1 is 1 shilling and 8 pence?
15. What part of £1 is 5s. 3d? Is 7s. 11d.? 15s. 7d.?
16. What part of 1s. is 4$\frac{1}{2}$d.? Is 6$\frac{1}{4}$d.? 10$\frac{1}{2}$d.? 11$\frac{3}{4}$d.?
17. What part of £1 is $\frac{2}{3}$ of a penny?
18. What part of a year is 4 months and 15 days?

Analysis.—4 mos. are $\frac{4}{12}$ or $\frac{8}{24}$ of a year, and 15d. are $\frac{1}{2}$ mo. = $\frac{1}{2}$ of $\frac{1}{12}$ year, or $\frac{1}{24}$ year, which added to $\frac{8}{24}$ year makes $\frac{9}{24}$ or $\frac{3}{8}$ year. Therefore, 4 mos. 15d. are $\frac{3}{8}$ year.

19. What part of a year is 5 mos. 6 d.? 7 mos. 5 d.?
20. What part of a year is 8 mos. 20 d.? 9 mos. 10 d.?
21. What part of a year is 4 mos. 5 d.? 3 mos. 6 d.?
22. What part of a year is 10 months and 25 days.

QUESTIONS FOR REVIEW.

97. Ex. 1. How much linen, at $1 per yard, can be bought for 12½ cents?

2. At £1 per hundredweight, how much cotton can be obtained for 3s. 4d.?

3. What part of a yard of bombazine can be obtained for 2½d., when it is 7s. per yard?

4. At £2 per ton, what part of a ton will 10s. buy?

5. If a lad can walk 3 miles in one hour, what distance can he walk in 20 minutes?

6. What will 7 gallons of oil cost, at $81 per hhd.?

7. At $10 per week, what will a person's board for 5 days come to?

8. If a man can perform a piece of work in 5 days, what part of it can he do in 3 days?

9. How many citrons, at 20 cents, can you buy for $5?

10. A man bought 40 cows, at $20 per head: how many double eagles did they come to?

11. If there are 96 grains of silver in 1 thimble, how many pennyweights are there in 12 thimbles?

12. At 5 cents a pound, how many dollars will 3 cwt. of flax come to?

13. If I pay $12½ per hundred, how much will 2 tons of maple sugar cost me?

14. What will 5 cwt. of rice cost, at ½ dime a pound?

15. What will 6 gal. of vinegar cost, at 1½ dime a qt.?

16. What will a hogshead of molasses come to, at 10 cents per quart?

17. A grocer paid $2 for a bushel of blackberries, and sold them at 10 cents a quart. How much did he gain or lose by the operation?

18. A man sold 3 pecks of chestnuts for 10 cents a quart, and took his pay in cider at 6 cents a pint: how many gallons of cider did he receive?

METRIC WEIGHTS AND MEASURES.

98. ***The Metric System of Weights and Measures*** is founded upon the *decimal notation*, and is so called because its *primary unit* is the *Me'ter*.

Long Measure.

99. The ***Me'ter*** is the *standard unit of length*, and is equal to 39.37 inches nearly.

The names of the *higher* denominations are formed by prefixing to the *unit* the Greek numerals, *dek'a, hek'to, kil'o*, and *myr'ia*; as, *dek'a-me'ter, hek'to-me'ter*, etc.

The names of the *lower* denominations are formed by prefixing to the *unit* the Latin numerals, *dec'i, cent'i*, and *mil'li*; as, *dec'i-me'ter, cen'ti-me'ter, mil'li-me'ter*.

NOTES.—1. The prefix *dek'a* means *ten*; hek'to, a *hundred*; *kil'o*, a *thousand*; and *myria*, ten thousand. Thus, a dekameter denotes 10 meters; a hektometer, 100 meters; a kilometer, 1000 meters, and a myriameter 10,000 meters.

2. The prefix *dec'i*, denotes *one tenth*; *cent'i*, one *hundredth*; and *milli, one thousandth*. Thus, a decimeter denotes $\frac{1}{10}$, or .1 of a meter; a centimeter $\frac{1}{100}$, or .01 of a meter; and a millimeter $\frac{1}{1000}$, or .001 of a meter.

3. The *numeral prefixes* are the *key* to the whole system and should be *thoroughly* mastered.

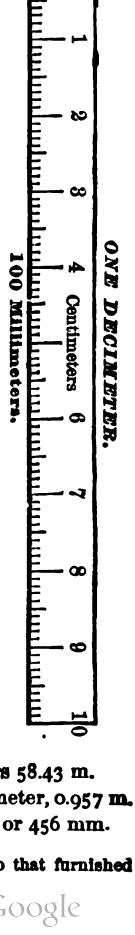
TABLE.

10 *mil'li-me'ters* (*mm.*)	make	1 cen'ti-me'ter,	*cm.*
10 cen'ti-me'ters	"	1 dec'i-me'ter,	*dm.*
10 dec'i-me'ters	"	1 ***Me'ter***,	*m.*
10 me'ters	"	1 dek'a-me'ter,	*Dm.*
10 dek'a-me'ters	"	1 hek'to-me'ter,	*Hm.*
10 hek'to-me'ters	"	1 *Kil'o-me'ter*,	*Km.*
10 kil'o-me'ters	"	1 myr'i-a-me'ter,	*Mm.*

4 The *meter*, like our *yard*, is used in measuring cloths, laces, moderate distances, etc.

For long distances, the *kilometer* is commonly used; and for minute distances, the *centimeter* and *millimeter*.

Ex. 1. How many meters in a Dekameter? In a Km.?
2. How many decimeters in a meter? Centimeters?
3. What part of a meter is 1 cm.?
4. What part of a meter is 1 mm.?
5. What part of a hektometer is 1 m.?
6. What part of a kilometer is 1 m.?
7. How many decimeters long is your desk?* How wide?
8. How many decimeters wide is the door?
9. How long is this room? How wide?
10. How long is the play-ground?
11. How many centimeters in 25 mm.?
12. How many meters in 376 cm.?
13. How many meters in 63 kilometers?
14. How many kilometers in 5260 meters?



100. The *accent* of each *unit* and *prefix* is on the *first* syllable, and remains so in the compound words.

To *abbreviate* the compounds, pronounce only the *prefix* and the first syllable of the unit; as, centim, millim, centil, decig, hectōg, etc.

101. Metric denominations are expressed in terms of a given unit, by writing the denominations *above* the unit in their order on the *left* of a decimal point, and those *below* the unit on the *right*, as decimals.

Thus, 5 Dm. 8 m. 4 dm. 3 cm. are written in meters 58.43 m.
Also, 9 cm. 5 dm. 7 mm. are written in terms of a meter, 0.957 m.
Again, 0.456 m. may be written 4.56 dm., 45.6 cm., or 456 mm.

* Every pupil should have a graduated metric rule, similar to that furnished by the Metric Bureau, Boston.

Square Measure.

102. The ***Unit*** for measuring ordinary *surfaces* is the *Square Meter*, which is equal to 1550 *sq. in.*

TABLE.

100 sq. cen'ti-me'ters (*sq. cm.*)	make	1	sq. dec'i-me'ter,	*dm.*
100 sq. dec'i-me'ters	"	1	***Sq. Me'ter***	*sq. m.*

NOTE.—The *square meter* is used in measuring flooring, ceilings, etc.; *square decimeters* and *centimeters*, for minute surfaces.

1 sq. centimeter = 100 sq. millimeters.

sq. cm.

The ***Unit of Land Measure*** is the *Ar*, and is equal to a *square dekameter*, or 119.6 *sq. yds.*

100 cent-ars (*ca.*)	make	1	***Ar***	*a.*
100 ars	"	1	hekt-ar	*Ha.*

Cubic Measure.

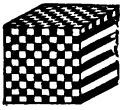
103. The ***Unit*** for measuring *ordinary solids* is the *Cubic Meter*, which is equal to 35.316 *cu. ft.*

TABLE.

1000 cu. mil'li-me'ters (*cu. mm.*)	make	1	cu. cen'ti-me'ter	*cu. cm.*
1000 cu. cen'ti-me'ters	"	1	cu. dec'i-me'ter	*cu. dm.*
1000 cu. dec'i-me'ters	"	1	***Cu. Me'ter***	*cu. m.*

NOTES.—1. The *cubic meter* is used in measuring embankments, excavations, etc.; *cubic centimeters* and *millimeters*, for minute bodies.

1 cu. centimeter = 1000 cu. millimeters.



cu. cm.

The ***Unit of Wood Measure*** is called the *Ster*, and is equal to a *cubic meter*, or 35.316 cubic feet.

10 dec'i-sters	are	1	***Ster***	*st.*
10 sters	"	1	dek'a-ster	*Ds.*

2. In France, *firewood* is commonly measured in a *cubical box*, whose *length, breadth, and height* are each 1 meter.

Dry and Liquid Measure.

104. The ***Unit of Dry and Liquid Measure*** is the *Li'ter* (*Leeter*), which is equal to a *cubic decimeter*, or 1.0567 wine quart.

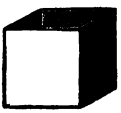
TABLE.

10 mil'li-li'ters (*ml.*)	are	1 cen'ti-li'ter - - -	*cl.*
10 cen'ti-li'ters	"	1 dec'i-li'ter - - -	*dl.*
10 dec'i-li'ters	"	1 ***Li'ter*** - - - -	*l.*
10 li'ters	"	1 dek'a-li'ter - - -	*Dl.*
10 dek'a-li'ters	"	1 *Hek'to-li'ter* - -	*Hl.*
10 hek'to-li'ters	"	1 kil'o-li'ter - - -	*Kl.*

NOTES.—1. The *liter* is commonly used in measuring milk, wine, etc., in moderate quantities. For minute quantities, the *centiliter* and *milliliter* are employed; and for large quantities, the *dekaliter*.

2. For measuring grain, etc., the *hektoliter*, which is equal to 2.8375 bushels, is commonly used.

3. The most convenient measures of capacity and weight for use in practical life are *doubles* of the unit and *halves* of the next higher denomination; as, the liter, double liter, half-dekaliter, etc.; the gram, the double gram, the half-dekagram, etc.



1 dry milli-liter.

1 liquid milli-liter.

4. Liters and Grams, with their multiples and subdivisions, are expressed in the same manner as meters. (Art. 101.)

1. How many centiliters in a liter? Deciliters? Milliliters?

2. How many liters make a hektoliter? A dekaliter? A kiloliter?

3. Write 5 hektoliters, 3 dekaliters, 4 liters, 2 deciliters, 8 centiliters, and 7 milliliters, in liters.

Weight.

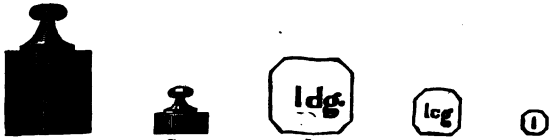
105. The ***Unit of Weight*** is the *Gram*, which is equal to 15.432 grains.

TABLE.

10 mil'li-grams (*mg.*)	make	1 cen'ti-gram - -	*cg.*
10 cen'ti-grams	"	1 dec'i-gram - -	*dg.*
10 dec'i-grams	"	1 ***Gram*** - - -	*g.*
10 grams	"	1 dek'a-gram - -	*Dg.*
10 dek'a-grams	"	1 hek'to-gram - -	*Hg.*
10 hek'to-grams	"	1 ***Kil'o-gram*** -	*Kg.*
10 kil'o-grams	"	1 myr'ia-gram - -	*Mg.*
100 myr'ia-grams	"	1 TONNEAU or Ton	*T.*

NOTES.—1. The *common unit* for weighing groceries and coarse articles is the *kilogram*, which equals 2.2046 lbs. Avoirdupois.

2. *Kilogram* is often contracted into *kilo*, and *tonneau* into *ton*.

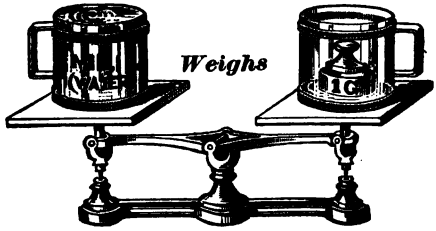


1 Dg. 1 Gram. 1 dg. 1 cg. 1 mg.

NOTES.—1. The *gram* and its multiples are made of *brass*; its subdivisions are made of *platinum*.

2. The *gram* is the weight used in Post Offices for weighing letters.

1. In a gram, how many milligrams? Centigrams?
2. How many grams in a kilogram? In a hektogram?



REMARK.—A milliliter, or cubic centimeter of distilled water is exactly equal to the brass weight called a *gram*.

Approximation of Metric Values.

106. When no great accuracy is required, consider

A decimeter	=	4 inches.	A hectar	=	$2\frac{1}{2}$ acres.
A meter	=	40 inches.	A ster (cu. m.)	=	$\frac{1}{4}$ cord.
5 meters	=	1 rod.	A liter	=	1 quart.
A dekameter	=	2 rods.	A hektoliter	=	1 bbl., or $2\frac{1}{8}$ bush.
An Ar	=	4 sq. rds.			
A kilometer	=	5 fur., or $\frac{5}{8}$ mile.	A gram	=	$15\frac{1}{3}$ grains.
			A kilogram	=	$2\frac{1}{5}$ pounds.
A sq. meter	=	$10\frac{3}{4}$ sq. ft.	A metric ton	=	2200 pounds.

107. Exercises in Metric Weights and Measures.

Ex. 1. A merchant sold 9 m. 8 dm. of silk to one customer, 10 m. and 7 dm. to another: how many meters did he sell to both?

Analysis.—9 m. plus 10 m. are 19 m., and 8 dm. plus 7 dm. are 15 dm., equal to 1.5 m., which added to 19 m. make 20 m. 5 dm., or 20.5 m., *Ans.*

2. How many meters in 23 m. and 4 dm.; added to 5 m. and 3 dm.?

3. If you cut 8 m. 4 dm. from a piece of muslin containing 35 m. 7 dm., how many meters will be left?

Analysis.—8 m. from 35 m. leave 27 m., and 4 dm. from 7 dm. leave 3 dm., which added to 27 m. make 27.3 m., *Ans.*

4. If from 350 kilos of sugar 125 packages of 1 kilo each are taken, how much will be left?

5. If 1 meter of broadcloth is worth $8, how much are 7 meters worth?

Analysis.—7 meters are worth 7 times as much as 1 meter, and 7 times $8 are $56. Therefore, etc.

6. If 1 hektar of land costs $15, what will 5 hektars come to?

7. What will 12 liters of milk cost, at 6 cts. a liter?

8. What cost 12 kilograms of flour, at 8 cts. a kilogram?

9. If 1 silver dollar weighs 25 grams, how much will 6 dollars weigh?

10. At 6 cents a liter, how many liters of chestnuts can you buy for 72 cents?

Analysis.—If for 6 cents you can buy 1 liter of chestnuts, for 72 cents you can buy as many liters as there are times 6 in 72, or 12 liters, *Ans.*

11. How many liters of vinegar, at 8 cts. a liter, will 96 cts. pay for?

12. If it cost me 12 cents to ride 1 kilometer, how far can I ride for 75 cents?

13. At $3\frac{1}{2}$ per Hl., what cost 7 hektoliters of wheat?

14. What cost 75 kilos of sugar, at 20 cts. a kilo?

15. How many metric tons of hay, at $10 a ton, can be had for $95?

16. How many square meters in a garden whose length is 20 m. and its breadth 8 m.?

17. What is the size of a Park 40 Dm. long and 25 Dm. wide?

18. Among how many persons can 80 kilos of flour be distributed, giving each 10 kilos?

19. If a druggist puts up a kilo of quinine in boxes of 1 gram each, how many boxes will be required?

20. At 10 cents a liter, what will a dekaliter of cranberries cost?

21. If you divide 2 hektoliters of corn among 100 persons, how much will each have?

[For a fuller illustration of the Metric System, see p. 153; also New Practical Arithmetic.]

☞ Mr. A. P. Marble, Supt. of Pub. Inst. of Worcester, Mass., has condensed the main points of the system into an epitome of two pages, forming a convenient pocket companion for reference.

CHAPTER X.

PERCENTAGE.*

Preliminary Exercises.

108. Ex. 1. When a number or thing is divided into 100 equal parts, what is one of these parts called? 2 of them? 6 of them? 20? 50?

2. What are hundredths sometimes called, and why?

Ans. Per cent, which means hundredths.

3. What is meant by 2 per cent? By 3 per cent? By 5 per cent? By 10 per cent?

4. To what is 100 per cent of a number equal?

Ans. To the *number* itself.

5. Why?

Ans. Because 100 *hundredths* make a *unit,* or 1.

6. What part of a number is 50 per cent?

Analysis.—50 is 1 *half* of 100; therefore 50 per cent of a number is 1 half of that number.

7. What part of a number is 25 per cent?
8. What part of a number is 20 per cent?
9. What part of a number is 10 per cent?
10. What part of a number is 5 per cent?
11. What part of a number is 2 per cent?
12. What part of a number is $12\frac{1}{2}$ per cent?
13. What part of a number is $33\frac{1}{3}$ per cent?
14. What part of a number is $16\frac{2}{3}$ per cent?
15. What part of a number is 100 per cent?

* The term Percentage is from *per* and *centum*, which means by the *hundred.*

109. Calculations in Percentage.

Ex. 1. How many parts or elements are to be considered in calculations of Percentage?

Ans. ***Four***, viz.: The *Base*, the *Rate per cent*, the *Percentage*, and the *Amount*.

2. Explain each of these parts.

Ans. The *base* is the *number* on which the *percentage* is calculated. The *rate per cent* is the *number* which shows *how many hundredths* of the *base* are to be taken. The *percentage* is the *number obtained* by taking that portion of the *base* indicated by the rate *per cent*. The *amount* is the *base plus*, or *minus* the *percentage*.

3. When it is said that 10 per cent of 30 is 3, which is the base? The rate per cent? The percentage?

4. When we say 8 is 20 per cent of 40, which number is the base? The rate per cent? The percentage?

110. Finding any required per cent of a number.

Ex. 1. What is $12\frac{1}{2}$ per cent of 72?

Analysis.—$12\frac{1}{2}$ per cent is 1 eighth of 100 per cent, and 100 per cent of a number is equal to the number itself. Therefore, $12\frac{1}{2}$ per cent of 72 is $\frac{1}{8}$ of 72, which is 9, *Ans.*

2. What is 20 per cent of 55?
3. What is 5 per cent of 100?
4. What is 10 per cent of 120?
5. What is 50 per cent of 140?
6. What is $33\frac{1}{3}$ per cent of 75?
7. What is 25 per cent of 80?
8. A man paid $200 for a horse, and sold it for 25 per cent above cost: how much did he gain?

Analysis.—25 per cent of a number is the same as $\frac{1}{4}$ of it. Now $\frac{1}{4}$ of $200 is $50. Therefore, he gained $50.

Or, thus: 25 is $\frac{25}{100}$ of a number. Now $\frac{1}{100}$ of $200 is $2, and $\frac{25}{100}$ are 25 times $2, which are $50, the same as before.

NOTE.—When the rate per cent is an aliquot part of 100 per cent, it is generally more convenient to use the aliquot part, than the number of hundredths.

9. A man paid $50 for a cow, and sold it at 10 per cent less than cost: what was his loss?

10. A jeweler bought a watch for $160, and sold it at an advance of 12½ per cent: what was his gain?

11. A lady paid $400 for a piano, and afterward sold it for 50 per cent of the sum paid: what was her loss?

12 Bought a village lot for $600, and sold it at a profit of 33⅓ per cent: what was the gain?

13. A merchant paid $250 for an India shawl, and sold it at 100 per cent profit: how much did he make?

14. A collector received 5 per cent for collecting a school tax of $200: how much was his commission?

15. A commission merchant charged his employer $800 for selling a lot of goods, but on settlement deducted 25% from his bill: how much did he deduct?

16. A broker charged ¼ per cent for buying railroad bonds amounting to $1000: how much was his brokerage?

17. At ½ per cent commission, what will be the brokerage for investing $2000 in bank stocks?

111. Finding the Amount of a number, when increased or diminished by a given per cent of itself.

Ex. 1. What is the amount of 45, when increased by 20 per cent of itself?

Analysis.—20 per cent is equal to ⅕ of a number, and ⅕ of 45 is 9. Now 45 and 9 are 54, *Ans.*

2. To what will 64 amount, if increased by 25 per cent of itself?

3. George has 60 marbles: how many will he have if he gains 5 per cent more?

4. What will 100 become if diminished by 10 per cent of itself?

5. If 72 is diminished by $12\frac{1}{2}$ per cent of itself, what will it become?

6. If 150 is increased by $33\frac{1}{3}$ per cent of itself, what will it become?

7. A man paid \$120 for a buggy, and sold it at 25 per cent above cost: what was the selling price?

Analysis.—The selling price was the cost plus 25 per cent of itself. Now 25 per cent of \$120=\$30, and \$120+\$30=\$150, *Ans.*

8. Harry solved 50 examples last week, and this week 20 per cent more: how many has he solved this week?

9. William paid 48 cents for a knife, and sold it at a loss of $12\frac{1}{2}$ per cent: what did he receive for it?

10. A merchant bought a bill of goods for \$400, and sold them at a profit of 5 per cent: what sum did he receive for the goods?

11. An orange grove produced 75 boxes of oranges last year, and this year $33\frac{1}{3}$ per cent less: how many boxes has it produced this year?

12. A farmer raised 200 bushels of wheat and 50 per cent more of corn than wheat: how much corn did he raise?

112. Finding what per cent. one number is of another.

Ex. 1. What per cent of 10 is 3?

Analysis.—3 is $\frac{3}{10}$ of 10, and $\frac{3}{10}$ reduced to hundredths $= \frac{30}{100}$. But $\frac{30}{100}$ are the same as 30 per cent.

NOTE.—The solution of this example requires two steps:

1st. To find what part one number is of another. (Art. 60.)

2d. To reduce these parts to *hundredths*. (Arts. 37, 70.)

2. What per cent of 25 is 7? Is 6? Is 9? Is 11?

3. What per cent of 9 is 3? Of 12 is 4?

4. What per cent of $12\frac{1}{2}$ is 3? Is 8? Is 7? Is 10?

5. 6 is what per cent of $16\frac{2}{3}$? Of $33\frac{1}{3}$? Of 50?

6. 12 is what per cent of 20? Of 25? 30? 50?

7. 15 is what per cent of 10? Of 20? Of 25?
8. 20 is what per cent of 10? Of 25? Of 50?
9. 50 is what per cent of 200? (Art. 112, note.)
10. 36 is what per cent of 300? 48 of 400? 72 of 800?

113. Ex. 1. What per ct. of itself is $\frac{1}{4}$ of a number?

Analysis.—$\frac{1}{4}$ is equal to $\frac{25}{100}$; therefore, $\frac{1}{4}$ of a number is equal to 25 per cent of the number.

2. What per cent of a number is $\frac{1}{2}$ of itself? $\frac{3}{4}$ of itself?
3. What per cent of a number is $\frac{1}{5}$ of itself? $\frac{2}{5}$ of itself?
4. What per cent of a number is $\frac{1}{5}$ of itself? Is $\frac{2}{5}$? $\frac{3}{5}$?
5. What per cent of a number is $\frac{3}{10}$? Is $\frac{8}{10}$? $\frac{7}{20}$?
6. What per cent of a number is $\frac{3}{20}$? Is $\frac{7}{20}$? $\frac{9}{20}$? $\frac{11}{20}$?
7. What per cent of a number is $\frac{7}{25}$? Is $\frac{13}{25}$? $\frac{17}{50}$? $\frac{33}{50}$?

114. Ex. 1. A grocer paid $25 for a chest of tea, and sold it for $6 more than he gave for it: what per cent was his profit?

Analysis.—The gain $6, is $\frac{6}{25}$ of $25, the money laid out. Now, $\frac{6}{25}$ equal $\frac{24}{100}$. Therefore, he made 24 per cent profit. (Art. 37.)

NOTE.—The point of inquiry in this and similar examples is, to find how many *hundredths* the gain is of the money employed.

2. Henry bought a sled for 50 cents, and sold it for 10 cents more than he gave: what per cent did he gain?

3. An apple-woman bought oranges at 4 cents apiece, and sold them so as to gain 2 cents on each: what per cent was her profit?

4. A merchant paid $33\frac{1}{3}$ for a case of goods, and sold it for $7 more than he gave: what per cent was his profit?

5. George sold his velocipede for $13, which cost him $10: how much did he make, and what per cent was his profit?

Analysis.—$13 less $10 = $3, the sum made. Again, $3 are $\frac{3}{10}$ of $10, and $\frac{3}{10} = \frac{30}{100}$. Therefore, he gained 30 per cent.

6. If I buy hats at $5 apiece, and sell them at $3, what is the per cent loss in cash?

7. If a man buys a cow for $25, and sells her for $50, what per cent is his profit?

8. Received a package of goods to sell on commission, the bill of which amounted to $50; if I charge $5 for my services: what per cent will be my commission?

9. A lawyer collected a note of $200, and charged $10 for his services: what per cent was his commission?

10. An auctioneer sold a private library for $500, and charged the owner $25 for his trouble: what was his per cent commission?

11. A broker invested $1000 in stocks, and charged his employer $10: what per cent was his brokerage?

12. If a broker charges $50 for selling $2000 of government bonds, what per cent will be his brokerage?

115. Finding a Number from a given per cent of itself.

Ex. 1. 40 is 25 per cent of what number?

Analysis.—25 per cent is equal to $\frac{1}{4}$. Now, since 40 is $\frac{1}{4}$ of a number, $\frac{4}{4}$ or the whole number must be 4 times 40 or 160, *Ans.*

Note.—The solution of this and similar examples is the same in principle, as finding a number when a *fractional* part of it is given.

2. 60 is 20 per cent of what number?

3. 12 is 5 per cent of what number?

4. 90 is 50 per cent of what number?

5. 20 is $12\frac{1}{2}$ per cent of what number?

6. 25 is $33\frac{1}{3}$ per cent of what number?

7. 30 is 6 per cent of what number?

8. A druggist made $60 on a chest of medicines, which was $33\frac{1}{3}$ per cent of the cost: what was the cost?

Analysis.—$33\frac{1}{3}$ per cent is equal to $\frac{1}{3}$. Now, as 60 is $\frac{1}{3}$ of a number, $\frac{3}{3}$ or the number must be 3 times 60, or $180, *Ans.*

9. A milliner made $30 on a bill of goods, which was 25 per cent of the cost: what was the cost?

10. A merchant made $50 on a lot of muslins, which was 20 per cent of the cost: what was the cost?

11. A jockey lost $50 in the sale of a horse, which was 12½ per cent of the sum he paid: what did the horse cost him?

12. A grocer lost $200 on a cargo of apples, which was 50 per cent of the cost: what did he pay for his apples?

13. A dressmaker sold a cloak for $25, which was 33⅓ per cent more than cost: what was the cost?

14. An apothecary sold a quantity of drugs for 75 cents, and thereby made 50 per cent: what was the cost?

15. A goldsmith sold a watch for $250, which was 60 per cent more than cost: what was the cost?

116. When the *Selling price* of goods and the per cent profit or loss are given, to find the *per cent* profit or loss if sold at a different price.

Ex. 1. A merchant sold a case of goods for $15, and thereby gained 25 per cent: what per cent would he have gained had he sold them for $18?

Analysis.—This example involves two inquiries:

First, what was the cost of the goods?

Second, what per cent would be the profit, if sold for 18?

1st. 25 per cent = ¼; and the cost = ⅘; now ¼ + ⅘ = ⅝; consequently, the selling price $15, is equal to ⅝ of the cost? Now, if $15 is ⅝ of a certain sum, ⅛ of that sum is ⅛ of $15, which is $3, and ⅘ is 4 times $3, or $12. Hence the cost is $12.

2d. If he had sold them for $18, his profit would have been $18 − $12 = $6. Now $6 (profit) is $\frac{6}{12}$ or ½ of $12 (the cost), and ½ = $\frac{50}{100}$ or 50 per cent. Therefore, if he had sold the goods for $18, he would have made 50 per cent.

2. A lad sold a pair of skates for 8 shillings, and lost 33⅓ per cent: what per cent would he have lost if he had got 9 shillings for them?

3. A man sold a bureau for $30, and made 20 per cent: had he sold it for $35, what per cent would have been his profit?

QUESTIONS FOR REVIEW.

117. Ex. 1. If you pay $24 for a load of apples, and sell them at 12½ per cent more than cost, how much will be your profit?

2. A fruit-dealer paid $8 for a box of oranges, and sold them at 10 per cent profit: how much did he make?

3. A man paid $30 for a cow, and sold her at an advance of 20 per cent: what was his profit?

4. A grocer bought a cask of wine for $84, and sold it at a profit of 25 per cent: what was his gain?

5. A boy paid 75 cents for a knife, and sold it at a loss of 33⅓ per cent: how much did he lose?

6. A jockey paid $126 for a horse, and sold it for 16⅔ per cent more than cost: how much did he make? How much did he get for the horse?

7. A merchant bought a package of shawls for $500, and sold them at 30 per cent profit: how much did he make on the package?

8. A man paid $3200 for a farm, and sold it at a loss of 12½ per cent: how much did he get for it?

9. A grocer exported provisions which cost him $1200, at 16⅔ per cent advance: how much did he receive for them?

10. An agent sold a horse for $200, and received 12½ per cent commission: how much did he receive?

11. What is the expense of collecting a tax of $500, at 6 per cent commission?

12. What is the commission on $600, at 12½ per cent?

13. What will it cost to collect a note of $300, if you pay 16⅔ per cent?

14. What will it cost to collect a bill of $1200, if you pay 25 per cent?

15. If a bankrupt pays 50 per cent of his debts, how much will a creditor receive on a note of $2000?

16. At $\frac{1}{2}$ of 1 per cent, what is the brokerage on a bill of exchange of $1000?

17. A real estate broker sold a house for $4500, and charged 1 per cent: what was his brokerage?

18. At $\frac{3}{4}$ per cent for selling stock, what will be the brokerage on $8000?

19. If a man pays $35 for a box of lemons, for what must he sell them to make 20 per cent?

20. If a man paid $45 for a sofa, for what must he sell it to gain 25 per cent?

21. If a merchant buys goods amounting to $840, at what price must he sell them to make $12\frac{1}{2}$ per cent?

22. For how much must a person sell a lot of groceries costing $600, in order to gain 15 per cent?

23. A man paid $250 for a carriage, and sold it at a loss of 10 per cent: how much did he get for it?

24. A shopkeeper paid 50 cents apiece for canes, and sold them, at 56 cents: what per cent did he make?

25. If a grocer buys eggs at 12 cents a dozen, and sells them for 13 cents a dozen, what per cent does he make?

26. If a miller buys corn at 30 cents a bushel, and sells it at 25 cents, what per cent is his loss?

27. If a merchant sells muslin at $37\frac{1}{2}$ cents per yard, which cost him 30 cents, what per cent does he make?

28. A farmer bought a pasture for $200, and sold it for $175: what per cent did he lose?

29. A man paid $4000 for $\frac{1}{16}$ of a ship, and sold it for $4200: what per cent did he make by the operation?

30. A man sold a wagon for $60, which was 25 per cent more than it cost him: what did he pay for it?

31. A tailor sold a coat for $18, and made 20 per cent on the cost: what was the cost?

32. A man sold a cow for $33, and lost 10 per cent on the sum he paid for her: what did she cost him?

INTEREST.

118. Calculating Interest for one or more years.

NOTE.—Calculations in interest are based on the principles of percentage, with the additional element of *time*. The rate per cent has reference to the interest of $1 for 1 year.

Ex. 1. What is the interest of $60 for 1 year, at 5 per cent?

Analysis.—5 per cent of a number is equal to $\frac{5}{100}$, or $\frac{1}{20}$ of that number. Now, $\frac{1}{20}$ of $60 is $3, *Ans.* (Art. 110, Note.)

Or, thus: 5 per cent of a number is $\frac{5}{100}$ of that number. Now 1 per cent of $60 is $\frac{60}{100}$, and 5 per cent is 5 times $\frac{60}{100}$, or $\frac{300}{100}$, which is equal to $3.

2. What is the interest of $50 for 1 year, at 6 per cent?

3. What is the interest of $60 for 1 year, at 7 per cent?

4. What is the interest of $150 for 1 year, at 4 per cent?

5. What is the interest of $90 for 1 year, at 8 per cent?

6. What is the interest of $96 for 1 year, at 12½ per cent?

Analysis.—12½ per cent is equal to $\frac{1}{8}$ of 100 per cent, and $\frac{1}{8}$ of $96 is $12, *Ans.* (Art. 77.)

NOTE.—When the given per cent is an aliquot part of 100, it is more expeditious and therefore advisable to use the aliquot part.

7. What is the interest of $250 for 1 year, at 10 per cent?

8. What is the interest of $125 for 1 year, at 4 per cent?

9. What is the interest of $150 for 1 year, at 10 per cent?

10. What is the interest of $108 for 1 yr., at 12½ per cent? What is the amount?

SUGGESTION.—First find the interest, then add it to the principal.

11. What is the amount of $300 for 1 yr., at 6 per cent?

12. What is the amount of $140 for 1 yr., at 5 per cent?

13. What is the amount of $120 for 1 yr., at 8 per cent?

119. *Five parts* or *elements* are to be considered, in calculating interest: The *Principal*, the *Interest*, the *Rate*, the *Time*, and the *Amount*.

14. Explain each.

Ans. The ***Principal*** is the money *lent*.

The ***Rate*** is the per cent *per annum*, or for 1 year.

The ***Interest*** is the percentage.

The ***Time*** is the period for which the principal draws interest.

The ***Amount*** is the sum of the principal and interest.

15. What is the interest of $75 for 2 years, at 7 per cent?

Analysis.—The interest of $75, at 1 per cent, is $\frac{75}{100}$, and at 7 per cent it is 7 times $\$\frac{75}{100}$, or $\frac{\$525}{100} = \$5.25$.

Again, if the interest of $75 for 1 yr. is $5.25, for 2 yrs. it is twice as much, and 2 times $5.25 = $10.50, *Ans.*

16. What is the interest of $270 for 2 yrs., at 5 per cent?

17. What is the interest of $150 for 3 yrs., at 6 per cent?

18. What is the amount of $200 for 6 yrs., at 10 per cent?

19. What is the amount of $400 for 7 yrs., at 12½ per cent?

20. What is the amount of $500 for 8 years, at 6 per cent?

120. Finding Interest for Months.

NOTE.—In calculating interest, a *month* is regarded as *one-twelfth* of a year. Hence, finding the interest for months is taking as many parts of one year's interest as there are like parts of a year in the given months.

Ex. 1. What part of a year is 6 months? 4 months? 3 months? 2 months?

2. What part of a year is 8 months? 9 months? 10 months?

3. What part of a year is 1 month? 5 months? 7 months? 11 months?

4. What is the int. of $300 for 4 months, at 6 per cent?

Analysis.—At 6 per cent the int. of $100 for 1 year is $6, and the int. of $300 for the same time, is 3 times as much, or $18.

Again, since 4 mos. are $\frac{1}{3}$ of a year, the interest for 4 months must be $\frac{1}{3}$ of 1 year's interest, and $\frac{1}{3}$ of $18 is $6.

5. What is the int. of $50 for 3 months, at 6 per cent?
6. What is the int. of $40 for 4 months, at 5 per cent?
7. What is the int. of $200 for 5 months, at 6 per cent?
8. What is the int. of $120 for 2 months, at 4 per cent?
9. What is the int. of $250 for 6 months, at 3 per cent?
10. What is the interest of $600 for 7 months, at 6 per cent?
11. What is the interest of $1200 for 9 months, at 4 per cent?
12. What is the interest of $1600 for 8 months, at 12½ per cent?
13. What is the amount of $300 for 6 months, at 8 per cent?
14. What is the amount of $500 for 3 months, at 6 per cent?
15. What is the amount of $1000 for 4 months, at 5 per cent?

121. Finding Interest for Days.

NOTE.—In computing interest, 30 days are commonly considered a month.

EX. 1. What is the interest of $600, at 10 per cent, for 15 days?

Analysis.—At 10 per cent, the interest of $600 for 1 year, or 12 months, is $\frac{1}{10}$ of $600, which is $60, and the interest for 1 month is $\frac{1}{12}$ of $60, or $5. (Art. 110.)

Again, 15 days are $\frac{1}{2}$ mo.; therefore, the int. of $60, at 10 per cent for 15 days, must be $\frac{1}{2}$ of $5, or $2\frac{1}{2}$, *Ans.*

2. What is the interest of $120 for 10 days, at 5 per cent?

3. What is the interest of $160 for 6 days, at 10 per cent?

4. What is the interest of $1200 for 8 days, at 4 per cent?

5. What is the interest of $3000 for 20 days, at 6 per cent?

122. Finding at what *Per cent* a given Principal will earn a given Interest in a given time.

EX. 1. At what per cent will $150 earn $18, in 2 years?

Analysis.—At 1 per cent the interest of $150 for 1 year is $1\frac{1}{2}$, and for 2 years it is 2 times $1\frac{1}{2}$, or $3. Now since $3 are 1 per cent interest on $150 for 2 years, $18 must be as many per cent as 3 is contained times in 18, or 6 per cent, *Ans.*

2. A man loaned $75 for 1 year, and received $10 interest: what was the per cent?

3. At what per cent must $100 be loaned to produce $8 interest in 3 years?

4. At what per cent will $200 gain $25 int. in $1\frac{1}{2}$ yr.?

5. At what per cent will $300 gain $50 interest in 4 years?

6. At what per ct. will $500 gain $100 int. in $5\frac{1}{2}$ years?

7. At what per ct. will $100 double itself in 10 years?

NOTE.—A principal is doubled when the interest earned is equal to itself.

Analysis.—At 1 per cent, the interest of $100 for 1 year is $1; and for 10 years it is $10. Now since $10 is 1 per cent interest for the given time, $100 interest must be as many per cent as $10 are contained times in $100, or 10 per cent, *Ans.*

Or, thus: at 100 per cent, any principal will double itself in 1 year. Now, if 1 year requires 100 per cent to double a principal, 10 years will require $\frac{1}{10}$ of 100 per cent, or 10 per cent.

8. At what per ct. will $125 double itself in 12 years?

9. At what per cent will $260 double itself in 5 years?

10. At what per cent will $826 double itself in 20 yr.?

11. At what per ct. will $1200 double itself in 15 yr.?

12. At what per cent will $2000 double itself in 8 years and 4 months?

123. Finding the *Time* in which a given Principal, at a given Rate, will earn a given interest.

Ex. 1. How long will it take $30, at 5 per cent, to earn $12 interest?

Analysis.—At 5 per cent, the interest of $30 for 1 year is $1½. Now since $1½ interest requires the use of $30 1 year, $12 interest will require the use of $30 as many years as $1½ is contained times in $12, which is 8 times. Therefore, to gain $12 interest, $30 must be loaned 8 years, at 5 per cent.

Or, thus: $12 is $\frac{12}{30}$ of $30; therefore, the interest is $\frac{12}{30} = \frac{4}{10}$, or 40 per cent of the principal. Now, since it requires 1 year to gain 5 per cent, to gain 40 per cent will require as many years as 5 per cent is contained times in 40 per cent, which is 8.

2. How long will it take $60 to gain $12 interest, at 8 per cent?

3. How long will it take $100 to gain $20 interest, at 6 per cent?

4. How long will it take $400 to gain $28 interest, at 7 per cent?

5. In what time will $500 gain $80, at 6 per cent?

6. In what time will $600 gain $120, at 10 per cent?

7. In what time will $1000 gain $200, when the rate is at 5 per cent?

8. In what time will $50 double itself, at 6 per cent?

Analysis.—Since at 1 per cent any principal requires 100 years to gain 100 per cent, or to double itself, at 6 per cent it will require $\frac{1}{6}$ of 100 years, or $16\frac{2}{3}$ years, *Ans.*

Or, thus: At 6 per cent. the interest of $50 for 1 year is $3. Now if $3 interest require a certain sum 1 year, $50 interest will require that sum as many years as $3 are contained times in $50, which is $16\frac{2}{3}$.

9. How long will it take for $63 to double itself at 7 per cent?

10. At 5 per ct., in what time will $100 double itself?

11. At 8 per ct., in what time will $320 double itself?

12. In what time will $500 double itself, at 10 per ct.?

13. In what time will $800 double itself, at $12\frac{1}{2}$ per ct.?

14. At 20 per cent, how long will it take for $1000 to double itself?

15. At 25 per cent, how long will it take for $1200 to double itself?

124. **Finding the *Principal*, when the interest, the rate per cent and the time are given.**

Ex. 1. What sum must be put at 6 per cent interest, to gain $25 in 5 years?

Analysis.—At 6 per cent, the interest of $1 for 5 years, is 30 cents, which is $\frac{30}{100}$, or $\$\frac{3}{10}$. Now, since $25 is $\frac{3}{10}$ of a sum, $\frac{1}{10}$ of that sum is $\frac{1}{3}$ of $25, which is $8\frac{1}{3}$, and $\frac{10}{10}$ are 10 times $8\frac{1}{3}$, or $83\frac{1}{3}$.

2. What principal, at 7 per cent, will gain $50 in 2 y.?

3. What principal, at 5 per ct., will gain $100 in 4 y.?

4. What principal, at $6\frac{1}{4}$ per ct., will gain $120 in 2 y.?

5. What principal, at 4 per cent, will gain $150 in 7 years and 6 months?

BANK DISCOUNT.

125. Finding Bank Discount.

NOTE.—Bank Discount is a deduction made by Banks, from the face of notes, drafts, etc., for advancing money on them. It is the same as simple int. paid in advance, for 3 d. more than the specified time. These are called days of grace.

Ex. 1. What is the bank discount on a note of $50, for 6 months, at 6 per cent?

Analysis.—At 1 per cent, the interest of $50 for 1 year, is $\frac{1}{100}$ of $50, and at 6 per cent it is $\frac{6}{100}$ or $\frac{3}{50}$ of $50, which is $3. Now, if the int. of $50 for 12 mo. is $3, for 6 mo. it is $\frac{1}{2}$ of $3, or 1\frac{1}{2}$, and for 3 d. ($\frac{1}{10}$ of 1 mo.) 2$\frac{1}{2}$ ct. (Art. 121.) Therefore, the bank discount is 1.52\frac{1}{2}$.

2. What is the bank discount on a note of $200, for 1 year, at 7 per cent?

3. What is the bank discount on a note of $120, for 4 months, at 5 per cent?

4. What is the bank discount on a note of $500, for 3 months, at 4 per cent; how much are the proceeds?

NOTE.—First find the *discount*, then subtract it from the note.

5. What are the avails of a draft of $200, on 6 months, at 5 per cent bank discount?

6. What are the avails of a note of $1000, for 60 days, at 5 per cent bank discount?

7. What is the bank discount on a draft of $200, payable in 4 months, at 7 per cent?

8. What is the bank discount on a draft of $350, for 30 days, at 6 per cent?

9. Required the bank discount on a note of $2000, for 60 days, at 6 per cent?

10. Required the avails of a draft of $150 for 3 months, at 8 per cent?

11. Required the avails of a draft of $1000 for 60 days, at 6 per cent?

EXCHANGE.

126. Computing Domestic Exchange.

NOTE.—*Exchange* is a method of making payments between distant places by means of *Drafts*, called *Bills of Exchange.*

Ex 1. A merchant in New Orleans wishes to remit $300 to New York: what will a draft cost him, at 2 per cent premium?

Analysis.—The draft will cost $300 plus 2 per cent of itself. But 1 per cent of $300 is $3, and 2 per cent is 2 times 3, or $6. Now, $300 + $6 = $306, *Ans.*

2. What cost a sight draft of $250 on San Francisco, at ¼ per cent premium?

3. What cost a sight draft of $500 on Philadelphia, at 4 per cent discount?

Analysis.—The draft is worth $500 minus 1 per cent of itself; but 1 per cent of $500 is $5, and 4 per cent is 4 times $5, or $20. Now, $500 − $20 = $480, *Ans.*

4. What is the worth of a draft of $350 on Chicago, at 3 per cent discount?

5. What is the worth of a bill of exchange on New York of $800, at 1½ per cent premium?

6. What cost a bill of exchange on St. Louis of $900, at 3 per cent discount?

7. Required the worth of a draft on Baltimore of $1200, at ½ per cent discount?

8. Required the value of a bill of exchange on Oregon, for $300, at 5 per cent discount?

9. Required the value of a sight draft on San Antonio, for $600, at 4 per cent discount?

10. What cost a bill of exchange on St. Louis, for $5000, at 2 per cent premium?

INSURANCE.

127. Finding the *Premium*,* when the sum insured, the rate per cent, and the voyage, or time are given.

Ex. 1. What premium must you pay for insuring books worth $350, from London to New York, at 2 per cent?

Analysis.—2 per cent. is $\frac{2}{100}$ or $\frac{1}{50}$ of the sum insured. Now, $\frac{1}{50}$ of $350 is $7. Therefore, etc.

2. What premium must a man pay on his furniture worth $500, at 1 per cent per annum?

3. What is the annual premium for insuring a house worth $2000, at $\frac{1}{2}$ per cent?

4. What is the annual premium on a vessel worth $6000, at 5 per cent?

5. A merchant insured silks worth $6000, from Canton to Boston: what was the premium, at $3\frac{1}{2}$ per ct.?

6. What is the premium on a policy† of Life Insurance for $2000, at 10 per cent per annum?

7. What is the premium on a life policy for $4000, at $12\frac{1}{2}$ per cent per annum?

8. Required the premium for insuring an interest of $5000 in the cargo of a whale ship to the end of her voyage, at 6 per cent?

9. What is the premium on an invoice of teas valued at $10000, from Canton to New York, when the rate of insurance is $2\frac{1}{2}$ per cent?

10. What must I pay a year for a policy of Life Insurance for $20000, when the premium is 10 per cent per annum?

* The *premium* is the sum paid for insurance.

† The *policy* is the contract containing the *terms* of insurance.

TAXES AND DUTIES.

128. Finding the *Tax*, the value of the property and the per cent being given.

Ex. 1. What is the tax on a house, the value of which is $1200, at 3 per cent?

Analysis.—At 1 per cent, the tax on $1200 is $12, and at 3 per cent it is three times $12, or $36, *Ans.*

2. If a man's property is valued at $2500, what will his tax be at 2 per cent?

3. A tax of 5 per cent was levied on a school district for educational purposes: what share must a man pay whose property is valued at $6000?

4. A tax of 4 per cent is voted by a township for improvement of roads: what would be a man's tax whose property was valued at $3000?

5. What is the specific duty on a chest of tea containing 120 pounds, at 12½ cents per pound?

Analysis.—12½ cents are $\frac{1}{8}$. Therefore, the duty will be $\frac{1}{8}$ as many dollars as there are pounds, and $\frac{1}{8}$ of 120 is 15. *Ans.* $15.

6. What is the duty on 150 yds. of silk at 50 cts. a yd.?

7. What is the duty on 200 hogsheads of molasses, at $1½ per hogshead?

8. What is the duty on 500 lbs. of coffee, at 6 cts. per lb.?

9. What is the ad valorem duty, at 25 per cent, on a quantity of goods invoiced at $2000?

Analysis.—25 per cent is equal to $\frac{25}{100}$, or $\frac{1}{4}$. And $\frac{1}{4}$ of $2000 is equal to $500, *Ans.*

10. What is the duty on a quantity of dry goods, valued at $4200, at 12½ per cent?

11. What is the ad valorem duty on an invoice of wines amounting to $6000, at 33⅓ per cent?

12. What is the ad valorem duty, at 40 per cent, on a cargo of tea valued at $12000?

CHAPTER XI.

GENERAL ANALYSIS.

FUNDAMENTAL PROBLEMS.

129. No specific directions can be given for the *analysis* of different classes of problems. The learner must depend on his *judgment* or *common sense* as a guide.

Beginners, however, may be aided by attending to the following general principles:

(1.) We reason from that which is *self-evident*, or *known*, to that which is *unknown*, or *required*.

(2.) We reason from a *part* to the *whole*; as, when the *value* of *one* is given, and the value of *two* or *more* of the same kind is required.

(3.) We reason from the *whole* to a *part*; as, when the value of two or more is given, and that of a part is required.

(4.) We reason from a given *cause* to its *effect*; as, when different combinations of numbers are given, to find the result.

(5.) We reason from a given *effect* to its *cause*; as, when the combinations of one number with others, and the final *result* are given, to find the *original* number.

130. **Two or more numbers being given, to find their Sum.**

Ex. 1. Given three numbers, 12, 9, and 15, to find their sum.

Analysis.—The whole is equal to the sum of all its parts. Now 12 and 9 are 21, and 15 are 36, the sum required. (Prin. 1.)

2. The parts of a certain number are 15, 7, and 9: what is the number?

3. A man bought a cow, and paying $28 down, agreed to pay the balance in two installments of $12 each: what was the price of the cow?

4. If a farmer makes 20 pounds of maple sugar on one day, 12 the next, and 15 the next, how many pounds of maple sugar does he make in 3 days?

131. The Less of two numbers and their Difference being given, to find the Greater.

5. The less of two numbers is 38, and their difference is 9: what is the greater?

Analysis.—If the less of two numbers is increased by their difference, it is self-evident the sum must be equal to the greater. Therefore, 38 plus 9 = 47, the greater number.

6. The less of two numbers is 27, and their difference is 15: what is the greater number?

7. A and B counting their money found that B had 59 cents, which was 25 cents less than A's money: how much had A?

8. The number of male inhabitants in a certain town is 935, and the number of females exceeds the number of males by 115: how many females does the town contain?

132. The Greater of two numbers and their Difference being given, to find the *Less*.

9. The greater of two numbers is 63, and their difference is 13: what is the less number?

Analysis.—Since the difference added to the less number equals the *greater*, it follows that the greater diminished by the difference must be equal to the less. Now 63 minus 13 leaves 50, the number required.

10. The greater of two numbers is 30, and their difference 12: what is the less? What is their sum?

11. Florence had 36 credits at the close of the week, Louise 12 less: how many credits had Louise? How many had both?

133. The Product of two numbers and one of the factors being given, to find the other Factor.

12. Given the product of two numbers, 63, and one factor, 9, to find the other factor.

Analysis.—Since 63 is a product of two numbers, one of which is 9, the other factor must be 63 ÷ 9, or 7, *Ans.*

13. If one of two factors is 11, and their product is 120, what must be the other factor?

14. The area of a rectangular grass-plot is 108 square yards, and its breadth 9 feet: what is its length?

15. A farmer being asked how many cows he had, replied that his neighbor had 11 cows, and if his own number were multiplied by that of his neighbor, the product would be 132: how many cows had he?

16. The area of a croquet-ground is 240 sq. yds., and its breadth is 12 yds.: what is its length? (Art. 87, Note.)

134. The product of three factors and two of the factors being given, to find the other Factor.

17. The solid contents of a stick of timber are 9600 cubic inches, its breadth is 4 inches and its thickness 3 inches: what is its length?

Analysis.—As the breadth is 4 inches and the thickness 3 inches, the product of which is 12 inches, the length must be 9600 divided by 12, and 9600 ÷ 12 = 800 cu. in. (Art. 88, Note.)

18. The product of 3 factors is 120, one of the factors is 5, and another 4: what is the third factor?

19. A pile of wood containing 128 cubic feet, is 4 ft. long and 4 ft. wide: what is the length?

20. The continued product of 4 numbers is 240, three of which are 6, 5, and 4: what is the fourth factor?

21. A father of four children being asked the age of the eldest, said the continued product of all their ages is 480 years, the 3 youngest are 2, 4, and 6 years old required the age of the eldest.

135. The Dividend and Quotient being given, to find the Divisor.

22. Given 108 the dividend, and 9 the quotient, to find the divisor.

Analysis.—The quotient shows how many times the divisor is contained in the dividend; therefore, if the dividend is divided by the quotient, the result must be the divisor, and $108 \div 9 = 12$, *Ans.* (Art. 129, Prin. 1.)

23. If a given divisor is 40, and the quotient 12, what is the dividend?

24. A farmer distributed 96 pears equally among 8 boys: how many did each receive?

25. John having 132 chestnuts, gave 12 to each of his classmates: how many classmates had he?

136. The Sum and Difference of two numbers being given, to find the Numbers.

26. Given the sum of two numbers 50, and their difference 12, to find the numbers.

Analysis.—If the sum of two numbers is diminished by their difference, it follows that *half* the remainder must be the less number. Now, $50 - 12 = 38$, and $38 \div 2 = 19$, the less number. Again, the difference added to the less number must be the greater, and $19 + 12 = 31$, the greater number. (Art. 129, Prin. 1.)

Or, thus: To half the sum add half the difference, and the result will be the greater. Now half of 50 is 25, and half of 12 is 6, which added to 25 makes 31, the greater; and $25 - 6 = 19$, the less number.

27. Two boys gathered 48 quarts of walnuts; one gathered 6 quarts more than the other: how many did each have?

28. Two riflemen together fired 84 shots at Creedmoor, one making 14 more shots than the other: how many shots did each fire?

29. Two pieces of broadcloth contain 56 meters, and the difference in their length is 8 meters: what is the length of each piece?

30. The sum of two numbers is 120, and half the difference is 10: what are the numbers?

31. If the sum of two numbers is 160, and a third of their difference 4, what are the numbers?

32. A man and a boy agreed to do a job of work for $60, on condition that the man's wages should be twice as much as the boy's: how much did each receive?

Analysis.—They both had $60, and since the boy had 1 part while the man had 2 parts, they both had 3 parts. Now $\frac{1}{3}$ of $60 is $20, the boy's share, and $\frac{2}{3}$ of $60 are $40, the man's share.

33. Two persons being 100 miles apart, traveled towards each other, one going 3 times as fast as the other: how far did each travel before they met?

34. Divide 72 into 2 such parts that if $\frac{1}{6}$ of the greater be added to the less, the numbers shall be equal: what are the numbers?

Analysis.—By the conditions, if 1 sixth is taken from the greater and added to the less, the numbers will be equal; therefore, the difference between the two numbers must be equal to 2 sixths of the greater. But the greater is $\frac{6}{6}$ of itself; hence the less must be equal to 4 sixths of it. Now 6 sixths and 4 sixths are 10 sixths, which are equal to 72.

Again, if 10 parts of a number are equal to 72, 1 part is 1 tenth of 72, which is $7\frac{2}{10}$; and 6 parts are 6 times $7\frac{2}{10}$, or $43\frac{2}{10}$, the greater; also 4 parts are 4 times $7\frac{2}{10}$, or $28\frac{8}{10}$, the less.

35. Two newsboys, A and B, counting the day's profits, found that both had made 84 cents, and if A would give B $\frac{1}{7}$ of his profits, they would have equal sums: what was each one's gain?

36. A father divided $100 between his son and daughter in such a manner that if the sister should give her brother $\frac{1}{10}$ of hers, they would have equal sums: how much did each have?

137. When a number and a series of combinations with other numbers are given, to find the result.

37. If 56 is divided by 7, the quotient multiplied by 8, the product diminished by 14, the remainder divided by 5, the quotient increased by 12, the sum diminished by 7, and the remainder multiplied by 4, what is the result?

Analysis.—Beginning with 56 and performing the several operations as indicated, we have 60 as the result. (Art. 129, Prin. 4.)

38. If to 27 we add 6, divide the sum by 11, multiply by 7, subtract 5, divide by 4, multiply by 12, add 8, divide by 9, add 25, what is the result?

138. When a series of combinations of one number with others, and the final result are given, to find the original number.

Ex. 1. What number is that, to which, if 7 be added, and three subtracted from the sum, the remainder multiplied by 5, and the product divided by 4, the quotient will be 15?

Analysis.—Since the quotient, or final result is 15, and the divisor 4, the dividend must be 15×4, or 60. Again, since one of the factors which produced 60 is 5, the other factor must be $60 \div 5$, or 12. And since the remainder, after subtracting 3, is 12, before the subtraction it must be $12 + 3$, or 15. Finally, since 15 is the sum after 7 is added to it, before the addition it must have been $15 - 7$, or 8, which is the original number.

NOTE.—The principle by which this and similar examples are solved, is to begin with the *final result*, and *reverse each operation*, as indicated by the problem.

2. What number is that, from which, if 5 be subtracted, and 8 added to the remainder, the sum is 24?

3. What number is that, which if multiplied by 6, and the product divided by 3, the quotient is 16?

4. What number is that, from which, if 8 be subtracted, the remainder divided by 4, the quotient multiplied by 7, and the product increased by 6, the sum is 34?

5. A lad with a small capital started out to seek his fortune; the first day he lost 20 cents, the second day he added 30 cents to the remainder, the third day he doubled the sum he then had, and the fourth day, after losing half the product, he had 60 cents left: what was his original capital?

SIMPLE PROPORTION.

139. Ex. 1. If 8 barrels of flour cost $40, how much will 5 barrels cost?

Analysis.—1 is 1 eighth of 8: therefore 1 barrel will cost 1 eighth as much as 8 barrels; and 1 eighth of $40 is $5. Now, if 1 barrel costs $5, 5 barrels will cost 5 times $5, or $25, *Ans.* (Art. 129.)

Or, fractionally, 5 barrels are the same part of 8 barrels as $40 are of the cost of 8 barrels. But 5 barrels are $\frac{5}{8}$ of 8 barrels; therefore, 5 barrels will cost $\frac{5}{8}$ of $40. Now, 1 eighth of 40 is $5, and 5 eighths are 5 times $5, which are $25.

2. If 7 lbs. of tea cost 42 shillings, what will 10 lbs. cost?

3. If 9 sheep are worth $27, how much are 15 sheep worth?

4. If 10 hektoliters of flour cost $60, what will 12 hektoliters cost?

5. Suppose 30 dekaliters of molasses cost $25, how many dollars will 7 dekaliters cost?

6. If a man earns 54 shillings in 6 days, how much can he earn in 15 days?

7. If 12 men can build 48 dekameters of wall in a day, how many dekameters can 20 men build in the same time?

8. A gentleman divided 90 shillings equally among 15 beggars: how many shillings did 7 of them receive?

9. Suppose 75 kilos of butter last a family of boarders 25 days, how many kilos will supply them for 12 days?

10. If 7 meters of cloth cost $30, how much will 9 meters cost?

11. If 10 bbls. of beef cost $72, what will 8 bbls. cost?

12. If 7 acres of land cost $50, what will 12 acres cost?

13. If you pay $35 for a hogshead of molasses, what must you pay for 27 gallons?

14. If 17 cords of wood are worth $51, how much is $\frac{4}{5}$ of a cord worth?

15. If $5\frac{1}{2}$ tons of coal cost $44, what is that per hundred?

16. How long will it take 20 men to do a job, which 14 men can do in $8\frac{1}{3}$ days.

17. If $\frac{3}{4}$ bbl. of flour cost $4\frac{1}{2}$, what will $\frac{7}{8}$ bbl. cost?

18. A can chop a cord of wood in 4 hours, and B in 6 hours: how long will it take both to chop a cord?

Analysis.—A can chop $\frac{1}{4}$ of a cord, and B $\frac{1}{6}$ of a cord in 1 hour. Hence, both can chop $\frac{1}{4} + \frac{1}{6}$ cord $= \frac{5}{12}$ cord in 1 hour. Again, if $\frac{5}{12}$ cord take both 1 hour, $\frac{1}{12}$ cord will take them $\frac{1}{5}$ hour, and $\frac{12}{12}$ will take them 12 times $\frac{1}{5}$ hour $= \frac{12}{5}$ or $2\frac{2}{5}$ hours, *Ans.*

19. A water-tank has two faucets, one of which can empty it in 20 minutes, the other in 30 minutes: how long will it take both to empty it?

20. If it will take 10 men $8\frac{1}{3}$ days to finish a piece of work, how long will it take 11 men to finish it?

21. If $\frac{7}{8}$ of an acre of land cost £$\frac{14}{16}$, what will $\frac{3}{4}$ of an acre cost?

22. A man had 25 apple trees in his orchard, and the number of his apple trees was to that of his peach trees, as 5 to 6: how many peach trees had he?

Analysis.—5 is $\frac{5}{6}$ of 6; hence the number of his apple trees is $\frac{5}{6}$ the number of his peach trees. The question then is this: 25 is $\frac{5}{6}$ of what number? If 25 is $\frac{5}{6}$ of a certain number, $\frac{1}{6}$ is 5, and $\frac{6}{6}$ is 6 times 5, which are 30, *Ans.*

23. A boy's age was 12 years, and his sister's age was to his own age as 7 to 3: what was her age?

24. A cistern has 3 stop-cocks, one of which will empty it in 5 hrs., another in 10 hrs., and the other in 15 hrs.: how long will it take all 3 to empty it?

25. If 12 men can dig a certain ditch in 15 days, how long will it take 8 men and 6 boys to dig it, allowing 3 boys to be equal to 2 men?

26. A reservoir has 2 pipes, one of which will fill it in 4 hrs., and the other will empty it in 6 hrs.: when empty, how long will it take to fill it, if both are running?

Analysis.—The inlet will fill $\frac{1}{4}$, and the outlet will discharge $\frac{1}{6}$ of it in 1 hour; hence, if both are running, $\frac{1}{4} - \frac{1}{6}$, or $\frac{1}{12}$ part of it will be filled in 1 hour. Now, if to fill $\frac{1}{12}$ of a reservoir requires 1 hour, to fill $\frac{12}{12}$, or the whole reservoir, will require 12 times 1 hour, or 12 hours, *Ans.*

27. A pound of tea lasts a man and his wife 6 weeks; when the man is absent, it lasts the wife 10 weeks: how long will it last the man?

28. If it requires 18 days for 12 journeymen to finish a house, how long will it take 10 apprentices to finish it, allowing 5 apprentices can do the work of 4 journeymen?

29. If the railroad to Brighton Beach carries 800 passengers in 4 trains, how many trains will be required to carry 8400 passengers?

30. A steamboat and a train of cars left New York for Albany; the boat had gone 20 miles when the cars started, but the cars go 7 miles while the boat goes 5 miles: how far must the cars go before they overtake the boat?

Analysis.—Since the boat goes 5 miles while the cars go 7, it will go $\frac{5}{7}$ as far as the cars do after they start; hence 20 miles are $\frac{5}{7}$ of the distance the cars must go to overtake the boat. Now, if 20 miles are $\frac{5}{7}$ of a certain distance, $\frac{1}{7}$ of that distance is $\frac{1}{5}$ of 20 m., which is 10 m., and $\frac{7}{7}$ or the whole distance, is 7 times 10 m., which are 70 miles, *Ans.*

31. Two men start from the same place at the same time, and travel in the same direction; one goes 9 kilometers an hour, and the other at the rate of 21 kilometers in 3 hours, how far apart will they be in 12 hours?

32. An express agent was sent from Boston to Washington, who traveled 12 miles an hour; 7 hours after another was dispatched, who traveled 20 miles an hour: how long before the latter overtook the former?

COMPOUND PROPORTION.

140. Ex. 1. If 5 men can chop 15 cords of wood in 2 days, how many cords will 6 men chop in 8 days?

Analysis.—If 5 men chop 15 cords in 2 days, 1 man will chop $\frac{1}{5}$ of 15 cords, which is 3 cords, and 6 men will chop 6 times 3, or 18 cords in the same time. Again, if 6 men chop 18 cords in 2 days, in 1 day they will chop $\frac{1}{2}$ of 18, or 9 cords, and in 8 days, 8 times 9, or 72 cords, *Ans.*

2. If 2 boys can earn $10 in 4 days, how long will it take 5 boys to earn $50?

3. If 4 men can saw 36 sters of wood in 2 days, how long will it take 6 men to saw 84 sters?

4. If 6 hektoliters of apples last 18 persons 3 months, how long will 10 hektoliters last 8 persons?

5. If it costs 8 persons $72 to board 3 weeks, how much will it cost 5 persons to board 7 weeks?

6. If it takes 12 men 6 days, working 8 hours a day, to finish a piece of work, how long will it take 4 men, working 12 hours a day?

7. If 9 horses eat 27 tons of hay in 3 months, how long will it take 6 horses to eat 24 tons?

8. If the interest of $200 for 6 months is $6, what will be the interest of $500 for 8 months?

9. If 5 barrels of flour are required to supply a garrison of 40 men 2 months, how many barrels will be required to supply 10 men 6 months?

PARTITIVE PROPORTION.

141. Ex. 1. A man divided 40 apples between two boys in such a manner that A's apples were to B's as 2 to 3: how many did each receive?

Analysis.—Since A had 2 apples as often as B had 3, it follows that A had 2 and B 3 as often as the sum of 2 and 3, or 5 is contained times in 40. And 5 is in 40, 8 times. Therefore, A had 8 times 2, or 16 apples, B had 8 times 3, or 24 apples.

Or, fractionally, A had 2 parts, and B had 3; but $2 + 3 = 5$; hence, A had $\frac{2}{5}$ and B $\frac{3}{5}$. Now, $\frac{2}{5}$ of 40 are 16 apples, and $\frac{3}{5}$ of 40 are 24 apples.

2. A father divided $56 between his two children, so that the sums they received were as 3 to 4: how much did each receive?

3. Divide 63 into two such parts, that one shall be to the other as 2 to 1.

4. Divide 35 into two such parts, that one shall be 3 times the other.

5. Two farmers together have 55 cows; but one has 4 times as many as the other: how many has each?

6. Two families, one containing 4 persons, the other 5, chartered a stage-coach for $54; what proportion ought each family to pay?

7. Divide 48 into two such parts, that one shall be $\frac{5}{8}$ of the other?

8. Divide 32 into two such parts, that one shall be $2\frac{1}{3}$ times the other.

9. A farmer bought a cow and calf for $42; the cow was worth $5\frac{1}{2}$ times as much as the calf: what was the value of each?

10. A father divided $100 among his 3 sons and 4 daughters, giving $\frac{1}{3}$ as much to a son as to a daughter: how much did each receive?

11. A lad bought 4 apples and 2 oranges for 80 cents, paying 3 times as much for an orange as for an apple: what did he give apiece for each?

12. What number added to 3 times itself, will make 48?

Analysis.—A number added to 3 times itself, will make 4 times that number. The question then is this: 48 is 4 times what number? Now, 48 is 4 times $\frac{1}{4}$ of 48; and $\frac{1}{4}$ of 48 is 12, *Ans.*

13. What number added to $\frac{1}{3}$ of itself, will make 36?

14. What number added to $\frac{4}{5}$ of itself, will make 45?

15. A man being asked how far he had walked, replied that he had traveled 64 kilometers, and had ridden twice as far as he had walked: how far had he walked?

16. A lad bought apples, pears and peaches, in all 70; the number of his apples was twice that of his pears, and the number of his pears was twice that of his peaches: how many of each did he buy?

17. Divide 48 into three such parts that the first shall be twice the second, and the third three times the second.

18. A drover had hogs, cows, and sheep, in all 100; there were twice as many cows as hogs, and twice as many sheep as hogs and cows: how many of each had he?

19. Divide 45 in 4 such parts, that the 4th shall be twice the 3d; the 3d twice the second; the 2d twice the 1st.

20. A man bought a hat, vest, and coat, for which he paid $60; for his vest he gave twice as much as for his hat; and for his coat 4 times as much as for his hat and vest: what did he pay for each?

21. A man being asked how much money he had in his purse, replied that $\frac{1}{2}$, $\frac{1}{5}$, and $\frac{1}{10}$ of it was equal to $40: how much had he?

Analysis.—$\frac{1}{2} + \frac{1}{5} + \frac{1}{10} = \frac{8}{10}$. The question then is this: $40 is $\frac{8}{10}$ of what sum? If $40 is $\frac{8}{10}$, $\frac{1}{10}$ is $\frac{1}{8}$ of $40, which is 5, and $\frac{10}{10}$ or the whole sum is 10 times $5, which are $50. Therefore, etc.

22. A lad having lost $\frac{1}{2}$ and $\frac{1}{4}$ of his marbles, found he had but 15 left: how many had he at first?

23. A third of the trees in a certain orchard are pear trees, $\frac{1}{5}$ are peach trees, and the remainder 21, are plum trees: how many trees are there in the orchard?

24. What number is that, $\frac{1}{2}$ of which exceeds $\frac{1}{3}$ of itself by 12?

25. What number added to $\frac{1}{3} + \frac{1}{4} + \frac{1}{6}$ of itself, will make 84?

26. A young lady being asked her age, replied that if to her age $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{5}$ of itself be added, the sum would be 39 years: what was her age?

27. What number is that, $\frac{1}{3}$ and $\frac{1}{4}$ of which is equal to $\frac{4}{5}$ of 35?

28. Two boys talking of their ages, one said he was 14 years old; the other said $\frac{1}{2}$ and $\frac{1}{7}$ of this was $\frac{3}{4}$ of his age: how old was the latter?

29. A farmer bought a cart and a plough for $81; the price of the cart was 8 times as much as that of the plough: what was the price of each?

30. It is required to divide 60 into 4 such parts, that the second shall be 2 times the first, the third 3 times the first, and the fourth 4 times the first.

PARTNERSHIP.

142. Ex. 1. Three men joined in a speculation and made $24. A put in $10, B $20, and C $30: what was each one's share of the profit?

Analysis.—$10 + $20 + $30 = $60, the sum invested. Hence, their respective shares of the profits were $\frac{10}{60}$ or $\frac{1}{6}$, $\frac{20}{60}$ or $\frac{1}{3}$, and $\frac{30}{60}$, or $\frac{1}{2}$. Now, $\frac{1}{6}$ of $24 is $4, A's share; $\frac{1}{3}$ of $24 is $8, B's; and $\frac{1}{2}$ of $24 is $12, C's share, *Ans.*

2. A and B formed a partnership; A furnished $40, and B $30; they gained $50: what was each one's share?

3. Two brothers joined in an adventure; the elder put in $60; the younger $40; they gained $38: what was the share of each?

4. A, B, and C enter into partnership; A puts in $400, B $300, and C $200; they gain $90: what is the share of each?

5. A, B, and C bought a lottery ticket, which drew a prize of $120; A paid $6, B $8, and C $10: what was each one's share of the prize?

6. Three lads, one of whom was 16 years old, another 12 years, and the other 8 years, found a purse containing $72, and agreed to share it according to their ages: how much did each receive?

7. Two newsboys formed a partnership; A put in $12, and B $8; they gained $96: what was each one's share of the gain?

BANKRUPTCY.

143. ***Bankruptcy*** is inability to pay indebtedness.

The ***Assets*** of a bankrupt are the property in his possession.

His ***Liabilities*** are his debts.

NOTE.—The *Net Proceeds* are the assets *less* the expense of settlement. They are divided among the creditors according to *their respective claims.*

EX. 1. A merchant failed owing A $160, and B $240; his assets amounted to $80: what dividend did he pay each creditor?

Analysis.—His liabilities were $160 + $240 = $400, and the net proceeds were $80; therefore, he could pay $\frac{80}{400} = \frac{1}{5}$ of his debts. Now, $\frac{1}{5}$ of $160 = $32, A's dividend, and $\frac{1}{5}$ of $240 = $48, B's dividend.

2. A man goes into bankruptcy owing one creditor $300, and another $500; his net assets are $200: what per cent and how much can he pay each creditor?

3. A bankrupt owes A $48, B $60, and C $72; he has only $90: how much can he pay each creditor?

4. A man died insolvent, owing $2500. His property was sold at auction for $1700: how much will his estate pay on a dollar?

5. How much can an insolvent debtor pay on a dollar, who has personal property amounting to $3500, and owes $5000?

ALLIGATION MEDIAL.

144. ***Alligation Medial*** is finding the *mean* value of mixtures.

1. A grocer mixed 5 lbs. of black tea worth 5 dimes a pound, with 4 lbs. of green tea worth 6 dimes a pound: what was the mixture worth a pound?

Analysis.—Since 1 lb. of black tea is worth 5 dimes, 5 lbs. are worth 5 times 5, or 25 dimes; and since 1 lb. of green tea is worth 6 dimes, 4 lbs. are worth 4 times 6, or 24 dimes. Now, 5 lbs. + 4 lbs. are 9 lbs., and 25 d. + 24 d. = 49d., and if 9 lbs. of tea cost 49 d., 1 lb. will cost $\frac{1}{9}$ of 49d., which is $5\frac{4}{9}$ dimes, *Ans.*

2. A man mixed 9 lbs. of sugar worth 6 cents a pound, with 12 lbs. worth 5 cents a pound: what is the worth of a pound of the mixture?

3. If a grocer mixes lard worth 8 cts., 10 cts., and 13 cts. a pound respectively, what is a pound of the mixture worth?

4. A farmer sells oats at 24 cts. a bushel, barley at 35 cts., and corn at 40 cts.: what is the average price per bushel?

5. A lady bought 3 yards of blue ribbon, at 20 cents, 4 yards of red, at 25 cents, and 2 yards of white, at 30 cents: what was the average price per yard?

6. A milkman having 2 quarts of cream, at 30 cents, and 15 quarts of milk, worth 6 cents a quart, mixed them with 3 quarts of water: what was a quart of the mixture worth?

ALLIGATION ALTERNATE.

145. ***Alligation Alternate*** is finding the ***proportional parts*** of a mixture having a given value.

Ex. 1. A farmer wishes to mix a quantity of peas worth 6s. a bushel, with corn worth 3s. a bushel, so that the mixture may be worth 4s. a bu.: what part of each must he take?

Analysis.—Since the peas are worth 6s. and the required mixture 4s. a bu., he will lose 2s. on every bu. of peas he puts in. And since the corn is 3s. and the mixture 4s. a bu., he will gain 1s. on every bu. of corn he puts in.

The question then is this: How much corn must he put in to make up the loss on 1 bu. of peas? Now, if it requires 1 bu. of corn to gain 1s., to gain 2s. will require twice as much, or 2 bu. Therefore, he must put in 2 bu. of corn to 1 bu. of peas.

NOTE.—The principle by which this and similar examples are solved, is that the *excess* of one article *above* the mean price of the mixture, counterbalances the *deficiency* of another article which is *below* it.

2. A man has lard worth 8 cents a pound and 12 cents a pound, and wishes to make a mixture worth 9 cents a pound: what part of each must he take?

3. A grocer has cider worth 15 cts. a gallon, and molasses worth 45 cts. a gallon: what proportion of each must he use to make vinegar worth 25 cts. a gallon?

4. A man buys oats at 40 cents a bushel, and corn at 75 cents a bushel, and wishes to make a mixture which he can sell at 60 cents a bushel: how much of each must he use?

5. A miller has two kinds of flour, worth $6 and $9 a barrel respectively; he wishes to make a mixture worth $8 a barrel: how much of each must he use?

CHAPTER XII.

POWERS AND ROOTS.

146. Raising a number to any required Power.

NOTES.—1. A ***Power*** is the *product* of a number multiplied into itself. Thus, $2 \times 2 = 4$; $3 \times 3 = 9$, etc., 4 and 9 are powers.

2. Powers are divided into different degrees; as first, second, third, fourth, etc.

3. The *second* power is called a ***Square***; the *third* a ***Cube***.

4. The *name* shows *how many times* the number is taken as a factor to produce the power.

Ex. 1. What is the 4th power of 2?

Analysis.—The *fourth* power denotes the product of a number into itself taken four times as a factor, and $2 \times 2 \times 2 \times 2 = 16$, the answer required.

2. What is the cube of 3?
3. What is the square of 4? The square of 5? 7? 6?
4. The square of 9? 8? 10? 11? 12?
5. What is the cube of 2? Of 4? 5? 10? 12?
6. What is the fourth power of 2? Of 3? 5? 10?
7. What is the fifth power of 2? The sixth power of 2?
8. What is the square of $\frac{2}{3}$? *Ans.* $\frac{4}{9}$.

NOTES.—1. A *Fraction* is raised to a power by *involving* both the *numerator* and *denominator*, or multiplying the fraction into itself.

2. ***Mixed numbers*** should be reduced to improper fractions, then raised to the required power.

9. What is the square of $\frac{1}{2}$? $\frac{1}{4}$? $\frac{3}{4}$? $\frac{2}{5}$? $\frac{5}{7}$? $\frac{8}{9}$? $\frac{11}{12}$?
10. What is the cube of $\frac{1}{4}$? Of $\frac{2}{3}$? $\frac{1}{5}$? $\frac{3}{4}$? $\frac{2}{5}$? $\frac{4}{10}$? $\frac{5}{12}$?
11. What is the square of $2\frac{1}{4}$?

SOLUTION.—$2\frac{1}{4} = \frac{9}{4}$, and $\frac{9}{4} \times \frac{9}{4} = \frac{81}{16}$. Therefore, the square of $2\frac{1}{4}$ is $\frac{81}{16}$.

12. What is the square of $1\frac{2}{3}$? $3\frac{1}{3}$? $4\frac{1}{3}$? $6\frac{2}{3}$? $2\frac{2}{3}$?

NOTE.—The square of a *mixed* number whose fractional part is $\frac{1}{3}$, may be found by multiplying the integral part by 1 more than itself, and to this result annex $\frac{1}{3}$.

13. What is the square of $2\frac{1}{2}$?

Analysis.—3 is one more than 2; now 3 times 2 are 6, and $6 + \frac{1}{4} = 6\frac{1}{4}$, *Ans.*

14. What is the square of $3\frac{1}{2}$? $4\frac{1}{2}$? $5\frac{1}{2}$? $6\frac{1}{2}$? $7\frac{1}{2}$? $8\frac{1}{2}$?

15. What is the square of $9\frac{1}{2}$? $10\frac{1}{2}$? $11\frac{1}{2}$?

147. Finding the Root of a number.

NOTES.—1. A ***Root*** of a number is one of its *equal factors.*

2. *Roots,* like powers, are divided into degrees; as, the square, or second root, the cube, or third root, the fourth root, etc.

3. The *name* shows into *how many* equal factors a number is required to be divided.

Ex. 1. What is the square root of 4?

Analysis.—Since the *square* of a number is the product of two equal factors, it follows that one of these equal factors must be the *square root* of that number. Now 4 is the product of 2×2. Therefore, 2 is the square root of 4.

2. What is the cube root of 27?

Analysis.—$27 = 3 \times 3 \times 3$; hence, the cube root of 27 is 3.

3. What is the square root of 16? Of 36? 25? 49?

4. What is the square root of 64? 81? 121? 144?

5. What is the cube root of 8? 64? 125? 1000?

6. What is the fourth root of 16? Of 81?

7. What is the fifth root of 32? The sixth root of 64?

8. What is the square root of 100? 121? 144? 196? Of 400? 900? 1600? 2500? 3600?

9. What is the cube root of $\frac{8}{27}$?

Analysis.—$\frac{8}{27} = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$; hence, the cube root of $\frac{8}{27}$ is $\frac{2}{3}$.

NOTE.—The *root* of a *fraction* is found by taking the root of both its terms.

10. What is the square root of $\frac{16}{49}$? $\frac{25}{36}$? $\frac{9}{64}$? $\frac{81}{100}$?

11. What is the cube root of $\frac{27}{64}$? Of $\frac{8}{64}$? $\frac{64}{125}$?

12. What is the square root of $2\frac{1}{4}$?

Analysis.—$2\frac{1}{4} = \frac{9}{4}$, and the square root of $\frac{9}{4} = \frac{3}{2}$, or $1\frac{1}{2}$, *Ans.*

13. What is the square root of $6\frac{1}{4}$? Of $12\frac{1}{4}$?

14. What is the square root of $30\frac{1}{4}$? Of $42\frac{1}{4}$?

148. Drill Exercises in Powers and Roots.

Ex. 1. To the square of 4, add 9; extract the square root; subtract 2; cube the remainder; divide by 3; square root; add 5; multiply by 8; cube root; square the root; divide by 2, and find the cube root of the quotient.

2. To the square root of 81, add 7; square root; multiply by 16; cube root; subtract 2; cube the result; divide by 2; square root; multiply by 8; square root; add 1; cube it; subtract 4; square root; add 5; square root; add 15: what is the result?

3. To the cube of 3, add 9; square root; add 3; square root; multiply by 9; cube root; multiply by 12; square root; divide by 2; square it; subtract 5; square root; multiply by 12; add 1; square root; multiply by 4; add 7; cube root; square it: what is the result?

4. Multiply the cube root of 64 by 4; extract the square root; multiply by 20; subtract 16; square root; add 1; square root; add 1; square root; cube it; multiply by 9; subtract 8; cube root; add 6; multiply by 10; square root; divide by 2; cube it; add 7; divide by 12; subtract 3; cube root; add 7; square root; subtract 3: what is the result?

5. Involve 3 to the fourth power; square root; multiply by 3; cube root; multiply by 20; add 4; square root; cube root; fifth power; subtract 5; cube root; multiply by 12; square root; multiply by 10; add 4; square root; multiply by 2; fourth root; multiply by 30; subtract 11; square root: what is the result?

6. Square 3; triple the result; cube root; double it; square it; take a fourth of it; square root; triple it; square it; extract the 4th root; triple it; halve it; add $4\frac{1}{2}$; square root: what is the result?

7. Square 2; cube the result; square root; cube root; involve to 4th power; square root; square root again; involve to 5th power; halve it; extract the 4th root; involve to 6th power; square root; multiply by 4; extract the 5th root; square it; double it; square it again; extract the 6th root; double it; square it: what is the result?

8. A lad has 121 marbles, which he wishes to arrange in the form of a square: how many must he put in a row?

9. What is the length of a side of a square room, which contains 169 square feet?

10. A man wishes to make a cubical cistern which shall contain 125 cu. ft.: what will be the length of one side?

11. How many feet long is the side of a cubical bin which contains 64 cubic yards?

12. A man having a rectangular field 16 rods long and 9 rods wide, exchanged it for one of equal area in the form of a square: how many rods long was the side of his new field?

13. A general having 2500 soldiers, formed them into a square: how many did he put in rank and file?

14. In a certain city there is a square park which contains 10 acres: how many rods in the length of one of its sides?

CHAPTER XIII.

APPLICATION OF WEIGHTS AND MEASURES.

149. Weight.

Ex. 1. A grocer bought 30 pounds of maple sugar, which he melted into cakes of 4 ounces each: how many cakes had he?

Analysis.—4 oz. are ¼ lb.; therefore, he had as many cakes as there are fourths in 30 lb., which are 120. *Ans.* 120 cakes.

Or, thus: in 1 lb. there are 16 oz., and in 30 lbs. 30 times 16, or 480 oz. Now 4 oz. are contained in 480 oz., 120 times. *Ans.* 120 cakes.

2. A dairyman had 108 pounds of butter, which he wished to pack in jars of 9 pounds each: how many jars would he require?

3. A butcher has 518 pounds of lard: how many kegs of 20 pounds each can he fill, and how much will he have over?

4. A goldsmith wishes to know how many rings weighing 5 pwts. each, he can make from 1 pound of gold.

5. A druggist had 5 oz. of opium which he made into pills of 4 grains each: how many pills did he have?

6. A bushel of oats weighs 32 pounds:* how many bushels are there in 160 pounds?

7. How many bushels of wheat in 720 pounds, allowing 60 pounds to a bushel?*

8. How many teaspoons, each weighing 16 pwts., can be made from 2 lbs. of silver?

* Laws of the State of New York.

9. The legal weight of a nickel 5-cent piece is 5 grams:* what is the weight of 75 of these coins?

10. The legal weight of a silver half dollar is $12\frac{1}{2}$ grams:* what is the weight of 40 half dollars?

11. The weight of a gold eagle is 258 grains:* what is the weight of 4 double eagles?

150. Ex. 1. A dairy-woman churned 40 pounds of butter, which she made into rolls of $3\frac{1}{3}$ pounds each: how many rolls did she have?

Analysis.—She had as many rolls as $3\frac{1}{3}$ lbs. are contained times in 40 pounds. Now, $3\frac{1}{3}$ is $\frac{1}{3}$ of 10; hence, multiplying both the divisor and dividend by 3, we have 120 to be divided by 10, and $120 \div 10 = 12$ rolls, *Ans.*

2. A miller had 500 pounds of flour, which he put into bags containing $33\frac{1}{3}$ pounds apiece: how many bags did he require?

Analysis.—Reasoning as before, and multiplying both the divisor and the dividend by 3, we have $1500 \div 100 = 15$ bags, *Ans.*

NOTE.—When the divisor is $3\frac{1}{3}$, $33\frac{1}{3}$, etc., we see by inspection that the quotient is readily found by multiplying the dividend by 3, and dividing the product by 10, 100, etc., as the case may require.

3. How many pounds of sal soda, at $3\frac{1}{3}$ cents a pound, can be had for 60 cents?

4. How many packages, each weighing $33\frac{1}{3}$ pounds, can be made of 250 pounds of coffee?

5. What will these packages come to apiece, at $33\frac{1}{3}$ cts. a pound?

6. A caterer has 4 bushels of strawberries which he wishes to put up in quart-cans how many cans does he require?

7. A poulterer has 240 eggs, which he sent to market in boxes containing 2 doz. each: how many boxes did he fill?

* By Act of Congress.

8. A farmer having 200 bushels of wheat, sent it to New York in bags containing $2\frac{1}{2}$ bushels apiece how many bags did he send?

9. A saloon-keeper bought 80 quarts of milk, at 28 cents a gallon: how many gallons did he buy, and what did the milk cost him?

10. A druggist had a gallon of cologne which he put into half-pint bottles, and sold it for 25 cents a bottle: how much did it come to?

11. A merchant had two pieces of muslin prints, one contained 45 yards and the other 72 yards, which he wished to divide into equal dress-patterns, each containing the greatest number of yards possible to be cut in equal patterns from the two pieces: how many yards could he put in a pattern?

Analysis.—Since the patterns are to be equal, and to contain the greatest number of yards possible, it follows that the number of yards in a pattern must be the greatest common divisor of 45 and 72, which is 9. Therefore, each pattern will contain 9 yards.

12. A man having two farms containing 35 and 56 acres respectively, wishes to divide them into equal fields which shall contain the greatest number of acres possible: how many acres can he put in each?

13. A tailor had two rolls of broadcloth containing respectively 36 and 63 yards, which he sold in equal pieces, each piece having the greatest number of yards possible: how many yards were in a piece, and how many pieces did he sell?

14. How long is the fence required to enclose a square field the sides of which are 45 rods?

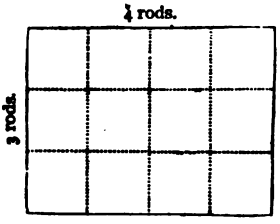
15. What is the length of a sidewalk 1 yard wide, which extends around a garden 30 yards long and 20 yards wide?

151. Measurement of Rectangular Surfaces.

Ex. 1. How many square rods in a garden 4 rods long and 3 rods wide?

ILLUSTRATION.—Let the adjoining figure represent the garden. There are as many *sq. rods* in the garden, as there are *squares* in the figure.

Now as there are 4 squares in 1 row, in 3 rows there must be 3 times 4, or 12 squares. Therefore, the garden contains 12 sq. rods.



2. How many sq. ft. in a hall 40 ft. long and 15 ft. wide?

3. How many yards of carpeting, 1 yd. wide, will it take to cover a floor, 21 feet long and 18 feet wide?

4. How many acres in a meadow 30 rods by 16 rods?

5. How many square yards in a grass-plot 36 feet by 20 feet?

6. How many yards of stuff, $\frac{4}{5}$ yd. wide, will it take to line a dress containing 8 yards of velvet, $1\frac{1}{4}$ yd. wide?

Analysis.—8 yards of velvet, $1\frac{1}{4}$ yd. wide = 10 sq. yds.
Again, 1 yard of stuff, $\frac{4}{5}$ yd. wide, is $\frac{4}{5}$ sq. yard, etc.

7. How many yards of Brussels carpeting, $\frac{3}{4}$ yd. wide, will be required to cover a floor 18 by 15 feet?

8. How many tiles, 8 by 9 inches, are required to pave a sidewalk 4 ft. wide and 25 feet long?

9. A man wishes to lay out a garden 20 rods long, containing 1 acre: what must be its width?

10. A wheat-field containing 5 acres, is 15 rods wide: what is its length?

11. A surveyor wishes to form a square field which shall contain $2\frac{1}{2}$ acres: what must be the length of its side in rods?

12. If the length of a lawn is 10 rods, what must be its width to contain $\frac{1}{4}$ of an acre?

152. Ex. 1. A lady planted a bed of tuberoses 6 feet long and 5 feet wide, placing the bulbs 1 foot apart and 6 inches from the sides of the bed: how many bulbs did she plant?

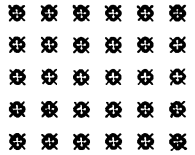


ILLUSTRATION.—Let a horizontal row of stars represent the number of plants in the length of the bed, and a perpendicular row the number in its width.

Since the outside rows were 6 inches from the edge, and the bulbs 1 foot from each other, it follows that each bulb occupied 1 square foot of ground. Hence she planted as many bulbs as there are stars in the diagram. Now as there are 6 bulbs in 1 row, in 5 rows there are 5 times 6, or 30. Therefore, she planted 30 bulbs.

2. How many strawberry vines can be planted in a bed 12 yds. long and 8 yds. wide, placing the vines 2 ft. apart and the outside rows 1 foot from the edge?

3. If a garden is 4 rods wide, how many rows of peas can be planted in it 1 yd. apart and 18 in. from the sides?

4. How many orange trees, which are 12 feet apart and the outside rows 6 feet from the fence, does a grove which is 132 feet long and 84 ft. wide, contain?

5. If each orange tree in this grove bears 2 boxes of oranges every year, and the boxes are sold at $3 a box, what will be the income from the grove per annum?

6. How many hills of potatoes can be planted in a field 65 yards long and 40 yards wide, allowing each hill occupies a square yard?

7. If each hill produces ½ peck of potatoes, how many bushels will the whole field produce?

8. If potatoes are worth ½ dollar a bushel, what is the value of the gross proceeds of the field?

9. What are the length and breadth of an orchard which contains 12 rows of apple trees and 9 trees in a row, the trees being 1 rod apart and ½ rod from the sides?

153. Measurement of Rectangular Solids.

Ex. 1. How many cubic feet are there in a rectangular block of granite 4 ft. long, 3 ft. wide, and 2 feet thick?

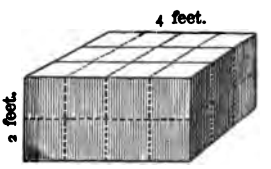


ILLUSTRATION.—Let the block be represented by the adjoining diagram. In the upper face there are 3 times 4, or 12 sq. ft. Now if the block were 1 foot thick, it would contain 1 time as many *cubic* feet as there are *square* feet in its upper face; and 1 time 4 into 3 = 12 cubic feet. But the given block is 2 ft. thick; therefore it contains 2 times 12, or 24 cu. ft., *Ans.*

2. A piano manufacturer ordered boxes made 8 feet long, 4 feet wide, and 1½ ft. deep, for packing instruments to send abroad: how many cubic feet did each contain?

3. A cistern 12 ft. long, 6 ft. wide, and 8 ft. deep, was constructed for the use of the Fire Department: how many cubic feet of water did it contain?

4. The block of stone of which the Cardiff giant was made, was 10 ft. long, 1 ft. wide, and 2 ft. thick: how many cubic feet did it contain?

5. A stick of timber 15 ft. long, is 2 ft. wide and 1½ ft. thick: how many cubic feet does it contain?

6. How many cubic feet in the two side walls of a railroad cut, 200 ft. long, 20 ft high, and 1½ ft. thick?

7. What are the cubical contents of a railroad car 40 feet long, 10 feet wide, and 8 feet high?

8. How many cubic feet in a stick of timber 50 feet long, 1½ feet wide, and 1 foot thick?

9. How many cubic inches in a plank 4 feet long, 12 inches wide, and 1½ inch thick?

10. How many cubic feet in a pile of wood 16 feet long, 4 feet wide, and 4 feet high? How many cords?

11. How many feet of wood in a pile 48 feet long, 4 feet wide, and 2 feet high? How many cords?

APPROXIMATE VALUES OF METRIC WEIGHTS AND MEASURES COMPARED WITH THOSE IN COMMON USE.

154. The object of the following exercises is to familiarize the learner with Metric Weights and Measures, by comparing their approximate value with those in present use. They should be supplemented by dictation until the object is gained.*

1. Draw a line a decimeter long.
2. Divide this line into centimeters.
3. Divide a centimeter into millimeters.
4. How many centimeters long is your slate?
5. How many wide?
6. In what, express the thickness of your slate?
7. In what, express the thickness of your reader?
8. The thickness of window-glass? Writing-paper?
9. How many decimeters in a meter? Centimeters?
10. What is the diameter of a five-cent piece? Of a fifty-cent piece? Of a silver dollar?
11. How many inches in 12 decimeters, allowing 4 inches to a decimeter? How many feet?
12. How many decimeters in a foot? In a yard?
13. Draw a line 1 meter in length. To how many millimeters is it equal? How many centimeters?
14. How many inches in 3 meters, allowing 40 inches to a meter? How many feet? How many yards?
15. How many inches in 5 meters? How many feet?
16. The length of this room in meters? The width?
17. How many meters wide is this street?
18. How many yards in a rod? Meters?
19. Mention an object that is 1 rod distant.
20. How many meters in 4 rods, or a surveyor's chain, allowing 5 meters to a rod? How many dekameters?

* See table of approximate values, p. 106.

155. Ex. 1. How many furlongs in a kilometer?

Analysis.—In 1 kilometer are 1000 meters. Now, if 5 meters make 1 rod, 1000 meters will make as many rods as 5 m. are contained times in 1000 m., or 200 rods. And since 40 rods make 1 furlong, 200 rods will make as many furlongs as 40 rods are contained times in 200 rods, or 5 furlongs, *Ans.*

2. What part of a mile is a kilometer?

Analysis.—A kilometer, we have seen, is equal to 5 furlongs. Now, since 8 furlongs make a mile, it follows that a kilometer is equal to $\frac{5}{8}$ of a mile, *Ans.*

3. In laying out a city, the surveyor makes 20 streets to ¼ mile: how many streets is that to a kilometer?

4. How many kilometers to 60 such streets?

5. How many kilometers in a league?

6. The distance from New Haven to Hartford is 30 miles: how many kilometers is it?

156. Measurement of Rectangular Surfaces.

1. Draw a square decimeter on the blackboard.

2. Divide one side of this into centimeters.

3. Draw a square meter on the blackboard or floor.

4. How many square centimeters in a square decimeter?

5. How many square decimeters in a square meter?

6. How many square meters in a square dekameter?

7. In measuring land, what is a square dekameter called?

8. What part of an *ar* is a square meter?

9. How many *ars* in a square hektometer?

10. By what *scale* do the metric units of square measure increase? *Ans.* By the scale of 100.

11. In writing metric numbers in square measure, how many *decimal* places must be allowed for each denomination? *Ans.* Two places.

12. Give an illustration upon the blackboard.

Thus, 34 sq. m. 5 sq. dm. 47 sq. cm. are 34.0547 sq. m.

157. Measurement of Rectangular Solids.

1. Draw a cubic centimeter on the blackboard.

2. Draw a cubic decimeter on the blackboard.

3. How many cubic centimeters make a cubic decimeter?

4. How many centimeters long, wide, and high, must a block of wood be to form a cubic decimeter?

5. How many decimeters long, wide, and high, must a box be to hold a cubic meter of grain?

6. How many cubic decimeters in a cubic meter?

7. What is a cubic meter called when used in measuring wood?

8. By what *scale* do the metric units of cubic measure increase? *Ans.* By the scale of 1000.

9. In writing metric numbers in cubic measure, how many decimal places must be allowed for each denomination? *Ans.* Three places.

10. Give an illustration upon the blackboard.

Thus, 374 cu. m. 62 cu. dm. 5 cu. cm. are written 374.062005 cu. m.

158. Measurement of Capacity.

What is the ***Primary Unit*** of dry and liquid measure?

Ans. The ***Liter.***

NOTE.—A *liter* is equal in *volume* to one *cubic decimeter.*

2. What is the *approximate* value of a liter?

3. What is the approximate value of 4 liters in liquid measure?

4. What is the approximate value of a hektoliter in dry measure?

5. In what metric denomination would you express the measurement of milk, wine, etc.?

6. In what would you express the measurement of grain, apples, etc.?

159. Weight.

1. What is the *primary unit* of weight?

Ans. The ***Gram,*** which is equal to 15½ grains.

NOTES.—1. The gram is determined by a careful measurement of pure water at its greatest density, which is the temperature of 4 degrees centigrade, or 39.2 degrees Fahrenheit.

2. The only units of weight used in practical life are the gram, kilogram, and ton. Jewellers, druggists, chemists, etc., use the subdivisions of the gram.

2. Which of our coins are determined by metric weights?

Ans. The silver half-dollar, quarter-dollar, the dime, and the nickle 5-cent piece.

3. What is the weight of a nickel 5-cent piece?

4. What is the weight of a silver half-dollar?

5. What is the approximate value of a kilo?

6. How many kilos in 110 pounds of sugar?

EXAMPLES FOR PRACTICE

1. How many meters in 3 remnants of cashmere, one of which contains 8.5 m., another 6½ m., a third 5.25 m.?

2. When silk is $2.00 per meter, what will 25 m. cost?

3. Bought carpeting at $2.50 a meter: what cost 60 m.?

4. If John walks to the store 1 Km., to the library ½ Km., to the depot 0.50 Km., to the church 2 Km., to the grocery 1½ Km., how many kilometers does he walk?

5. Four brothers measure in height as follows: A, 110 cm.; B, 120 cm.; C, 130 cm.; D, 150 cm.: how many centimeters do they all measure? How many meters?

6. What length of wood will be required for ten rulers, each 40 cm. long?

7. If a train of cars runs 60 Km. an hour, what distance does it run in 24 hours?

8. How many square meters in a sidewalk 4 meters wide and 65 meters long?

9. How many square meters in a courtyard 100 decimeters wide, 200 decimeters long?

10. How many square meters in the floor and ceiling of a room 50 centimeters wide and 60 centimeters long?

11. A man walked from Albany towards Buffalo, six days: Monday he walked 20 Km.; Tuesday, 25 Km.; Wednesday, 26 Km.; Thursday, 28 Km.; Friday, 29 Km.; Saturday, 30 Km.: how far did he walk in six days?

12. How many miles did the man walk in the six days, allowing a kilometer to be $\frac{5}{8}$ of a mile?

13. A milkman sold 40 liters of milk to one customer, 25 liters to another, 15 liters to a third, a double liter to a fourth, and a half-liter to a fifth: how much did he sell to all?

14. How many gallons of milk did he sell, allowing a liter is equal to a quart?

15. At 8 cents a liter, what will be the milk bill of a family which uses 5 liters of milk a day for four weeks, Sundays excepted?

16. At 12 cents a liter, what cost a dekaliter of walnuts?

17. What cost 4 kilos sugar, at 16 cents a kilo?

18. What cost 12 kilos of rice, at 10 cents a kilo, 5 Kg. beef, at 40 cents, 10 Kg. coffee, at 60 cents a kilo?

19. An express wagon carries 50 Kg. flour, 25 Kg. buckwheat, 10 Kg. tea, 20 Kg. sugar. What is the weight in the wagon?

20. From 150 kilos of sugar are taken 30 packages, of 2 kilos each. How much is left?

21. Among how many persons can 140 kilos of flour be distributed, giving each 10 Kg.?

22. A merchant divides 60 meters of broadcloth among a number of his clerks, giving to each 150 centimeters. To how many clerks does he distribute it?

23. A druggist put up a half-kilo of opium in boxes of 1 gram each. How many boxes did he need?

24. From a piece of linen 50 decimeters long, how many towels, each 50 centimeters long, can be cut?

25. How many square meters of tin will it take to cover the roof of a house 20 meters long, $8\frac{1}{2}$ meters wide?

26. What will the tinning of this roof cost, at $33\frac{1}{3}$ cents per square meter?

27. How many sq. meters in the walls of a room 12 m. long, 8 m. wide, and 6 m. high?

28. Required the cost to paint these walls, deducting the window 3 m. high, 1 m. wide, at $33\frac{1}{3}$ cents a sq. meter?

29. A dealer sold a carpet for the same room: how much did it come to, at $3.00 a sq. meter?

30. How much carpet-lining, $\frac{1}{2}$ m. wide, would be required for this carpet?

31. A druggist has a measure which holds 1 cubic decimeter of water. How many measures, each holding 10 cu. centimeters of water, would be required to hold the water of the cu. decimeter?

32. In a wood-yard are 50 piles of wood, of 3 Dst. each: how many piles can be made of 1 ster each?

33. At $2.00 a ster, what would be the value of the wood in the yard?

34. An apothecary uses from a glass holding a cubic decimeter of distilled water, one cubic centimeter every hour. How much does he use in 12 hours? How many cubic centimeters are left?

35. How many cubic decimeters in a rectangular stick of timber, 4 decimeters square and 50 decimeters long?

36. How many cubic meters in a cellar 20 meters long, 8 meters wide, $2\frac{1}{2}$ meters deep?

37. The sides of a rectangular reservoir are each 50 meters, and its depth is 8 meters: how many cubic meters of water does it contain.

TEST EXAMPLES FOR REVIEW.

Ex. 1. What number is that to which, if 16 be added, the sum will be $37\frac{1}{2}$?

2. A farmer has 43 acres in two pastures, one of them contains 9 acres more than the other: how many acres in each?

3. Two lads talking of their money, A said he had 46 cents, B said his exceeded 100 cents as much as A's fell short: how many cents had B?

4. A man being asked how far he had traveled, replied if he should go 63 miles more, the distance would be 100 miles: how far had he traveled?

5. A market-woman being asked how she sold apples, replied, 5 for 4 cents: how much was that a dozen?

6. A lad having 50 peaches, sold them at the rate of 2 for 5 cents: how much did he receive for them?

7. What cost $6\frac{1}{2}$ yards of ribbon, at $6\frac{1}{2}$ cents a yard?

8. What cost $7\frac{1}{2}$ lbs. of raisins, at $7\frac{1}{2}$ cents a pound?

9. A market-woman bought 12 oranges at $5\frac{1}{2}$ cents apiece; she sold $\frac{1}{2}$ of them for 40 cents, and the other half at $6\frac{1}{4}$ cts. apiece: how much did she make by the operation?

10. William has 45 hens and chickens, and 15 more chickens than hens: how many has he of each?

11. At an election of a school trustee, the whole number of votes cast was 51, and A had a majority of 11 over B: how many votes had each?

12. If 5 barrels of flour last 16 persons 8 months, how long will it last 1 person?

13. A man agreed to do a job in 1 day which would require 20 men 6 days: how many men must he employ?

14. A dairyman has 50 lbs. of butter, which are $5\frac{1}{2}$ times as much as he has sold: how much has he sold?

15. A lad has $\frac{3}{4}$ of 40 apples, which are $3\frac{1}{3}$ times as many as he gave away: how many did he give away?

16. If 5 men can do a piece of work in 18 days, how long will it take 1 man to do it?

17. When rice is $16\frac{2}{3}$ cts. a pound, how much can you buy with 50 cts.?

NOTE.—$16\frac{2}{3}$ cts. are one sixth of a dollar. (Art. 77.)

18. A laundress laid out \$1.50 in starch, at $16\frac{2}{3}$ cts. a pound: how much did she get?

19. A dairy-woman received \$4 for cheese, which was $16\frac{2}{3}$ cts. a pound: how many pounds did she sell?

20. A man paid \$2 for a buggy to go to town, which was $16\frac{2}{3}$ cts. a mile: how many miles was it?

21. How many strawberries, at $12\frac{1}{2}$ cents a quart, can be obtained for \$15?

22. What cost $6\frac{1}{2}$ yards of edging, at $16\frac{2}{3}$ cents a yard?

23. What cost $13\frac{1}{2}$ cords of wood, at $\$3\frac{1}{2}$ a cord?

24. If the area of a rectangular field is 89 square rods, and its length is 11 rods, what is its breadth?

25. A young man, after spending $\frac{1}{3}$ of his money and \$5, found he had \$20 left: how much had he at first?

26. It requires 100 sq. yds. of carpeting to cover the floor of a concert hall 24 feet wide: what is the length of the hall?

27. Thomas bought 36 apples for 25 cts., and sold them at the rate of 4 for 3 cts.: what did he make or lose?

28. If he had sold them at the rate of 3 for 2 cents, what would have been the result of his bargain?

29. The product of A, B, and C's ages is 240 years; A is 6 years, and B is 5 years old: how old is C?

30. The length of a pile of wood containing 200 cubic feet, is 10 feet, and its breadth is 4 feet: what is its height?

31. A man paid $110 for a horse and saddle; the horse was worth 5½ times as much as the saddle: what was the value of each?

32. What cost 4½ tons of coal, at $8½ a ton?

33. How many bushels of apples, at 30 cents, will it take to pay for 7½ yards of calico, at 16 cents a yard?

34. What is the product of 8½ multiplied by 8½?

35. A grocer sold 8 lbs. of sugar at 12 cents a pound, and took his pay in butter, at 10 cents a pound: how much butter did it take to pay for the sugar?

36. How many dozen eggs, at 15 cts. a dozen, will it take to pay for 12½ yds. of muslin, at 8 cents a yard?

37. At $6½ a yard, what will be the cost of 30 yards of velvet?

38. What cost 7½ bushels of wheat, at 90 cents a bushel?

39. At 12½ cents a dozen, what will 6½ dozen eggs cost?

40. What cost 40 tons of hay, at $10½ per ton?

41. How many acres of land worth $12½ per acre, must be given for 4 city lots, valued at $750 per lot?

42. How much lard, at 7 cents a pound, will it take to pay for 3⅓ lbs. of rice worth 4½ cents a pound?

43. How many oranges, at 7½ cents apiece, can you buy for ⅓ of 35 quarts of strawberries, at 12½ cents a quart?

44. How many pears, at 3⅓ cents, can be bought for 90 cents.

45. At 3⅓ dimes apiece, how many citrons can be bought for 25 dimes?

46. If you walk 3⅓ miles an hour, how long will it take you to walk 18 miles?

47. If you pay $3⅓ per week for board, how long can you board for $150?

48. If a man spends 33⅓ cents a day for brandy, how long will he be in spending $50?

49. How many pounds of coffee, at 33⅓ cts. a pound, will $25 buy?

50. If flannel is 33⅓ cts. a yard, how many yards can you buy for $120?

51. At 12½ cents each, what will 200 writing-books cost?

52. How much will 60 caps cost at 33⅓ cents apiece?

53. A farmer sold 72 pounds of butter at 16⅔ cents a pound: what did he receive for his butter?

54. At 33⅓ cents a gallon, what will 150 gallons of milk come to?

55. A man bought a cow for $35 and sold her for 10 per cent above cost: how much did he make?

56. A merchant paid $200 for a piece of silk, and sold it at 12½ per cent profit: how much did he gain?

57. A lady paid $300 for a parlor organ, and afterwards sold it at a loss of 20 per cent: what was her loss?

58. A lad bought a velocipede for $15, and sold it at 33⅓ per cent above cost: what did he get for it?

59. George had 75 apples, and gave away 20 per cent of them: how many had he left?

60. A jockey bought a horse for $360, and sold it at 25 per cent above cost: what was the selling price?

61. A merchant sold a bill of goods for $450, and thereby lost 15 per cent: what did the goods cost him?

62. If a newsboy buys papers at 2 cents apiece and sells them at 4 cents, what per cent is his profit?

63. A man bought a sofa at auction for $60, and sold it for $150: what per cent was his profit?

64. A tailor sold a suit of clothes for $50, which was 20 per cent above cost: what was the cost?

65. A milliner sold hats at $15 apiece, which was 33⅓ per cent above cost: what per cent would she have made if she sold them at $18?

66. A grocer lost $150 on a cargo of fruit, which was 50 per cent of the cost: what was the cost?

67. A miller sold flour at $6 a barrel and thereby lost 10 per cent: what per cent would have been his loss if he had sold it at $5 a barrel?

68. What is the interest of $70 for 1 year, at 5 per cent?

69. What is the interest of $150 for 3 years, at 6 per cent?

70. What is the interest of $400 for 4 months, at 7 per cent?

71. What is the amount of $500 for 3 months, at 6 per cent?

72. What is the amount of $600 for 7 months, at 8 per cent?

73. What is the interest of $120 for 10 days, at 6 per cent?

74. What is the interest of $2000 for 15 days, at 6 per cent?

75. What is the bank discount on a note of $150 for 4 months, at 6 per cent?

76. What is the bank discount on a note of $200 for 90 days, at 7 per cent? What are the avails?

77. What are the proceeds of a draft of $500 for 30 days, at 6 per cent bank discount?

78. If 6 oranges cost 42 cents, what will 8 oranges cost at the same rate?

79. If 9 pears cost 54 cents, what will 15 pears cost?

80. If 3 barrels of flour are worth $24, what are 70 barrels worth?

81. If 6 tubs of butter cost $42, what will 40 tubs cost?

82. If 8 yards of silk cost $32, what will 19 yards cost?

83. If 9 barrels of cider cost $45, what will 4 barrels cost?

84. At the rate of 7 lemons for 35 cents, what will 3 dozen lemons come to?

85. A grocer sold 5 melons for 70 cents: what was that per dozen?

86. What will my board amount to in 12 weeks, if I pay at the rate of $35 for 7 weeks?

87. If 7 pounds of flour cost 63 cents, what will 31 pounds cost?

88. If 4 men can do a piece of work in 6 days, how long will it take 8 men to do the same work?

SUGGESTION.—8 men are twice as many as 4 men; therefore 8 men will do it in *half* the time of 4 men.

89. If it requires 3 carpenters to finish a house in 120 days, how long will it take 24 to finish it?

90. If 2 faucets will fill a cistern in 24 hours, how long will it take 6 faucets to fill it?

91. If 12 men can dig a trench in 10 days, how long will it take 20 men to dig it?

92. If 1 man can lay 4 rods of wall in a day, how many rods can 3 men lay in 9 days?

93. How many cords of wood will 5 men saw in 90 days, each man sawing 2 cords per day?

94. A man had a certain job which 7 men could do in 12 days: how long would it take 10 men to perform the same job?

95. A man had a house to finish, which 8 men could do in 9 days, but he wished to have it completed in 6 days: how many men did it take to do it?

96. If the sum of two numbers is 60, and the greater is 3 times the less, what are the numbers?

97. What number is that to which, if $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ of itself be added, the sum will be 75?

98. What number is that, which added to 4 times itself, the sum will be 100?

99. If $\frac{1}{6}$ of a pole stands in the mud, $\frac{2}{3}$ of it in water, and 12 feet are above water, what is the length of the pole?

100. A man bought a piano, and paid $150 down, which was $\frac{3}{8}$ the price of it: what was the cost?

101. What number must be multiplied by $6\frac{1}{4}$, that the product may be $37\frac{1}{2}$?

102. What number must be multiplied by $2\frac{1}{2}$, that the product may be $15\frac{3}{4}$?

103. A drover having disposed of 55 sheep, which was $\frac{5}{8}$ of his whole number, for $100, sold the remainder at $3\frac{1}{2}$ a head: what did they all come to?

104. What will $5\frac{1}{4}$ lbs. of chloride of lime cost, at the rate of $10\frac{1}{2}$ dimes for $3\frac{1}{2}$ lbs.?

105. How many muffin rings, at the rate of 5 for 9 cents, can you buy for $\frac{5}{7}$ of 63 cents?

106. A farmer being asked how many sheep he has, replied, that his neighbor has 100, which is 9 times $\frac{5}{8}$ of his number: how many has he?

107. A lad being asked how old he was, answered that 48 years were 6 times $\frac{4}{7}$ of his age: how old was he?

108. If $\frac{5}{8}$ of a bushel of apples cost $\frac{1}{2}$ of $\frac{2}{3}$ of a dollar, what will $\frac{3}{4}$ of $\frac{4}{5}$ of a bushel cost?

109. If 5 men can dig a ditch 20 rods long in 8 days, how long will it take 8 men to dig one 12 rods long?

110. What number is that to which, if $\frac{1}{4}$ of itself plus 27 be added, the amount will be double the number?

111. Nine-tenths of a certain number exceeds $\frac{4}{5}$ of it by 6: what is the number?

112. If a herring and a half cost a penny and a half, how many can you buy for 11 pence?

113. A manufacturer being asked how many persons he employed, said 50 men, twice as many boys as men, and thrice as many girls as boys: how many persons did he have in his employment?

114. A can dig a cellar in three weeks, B in 4 weeks, and C in 6 weeks: how long will it take all three to dig it?

115. If a sleigh cost a man $60, what part of itself must he add to the cost to make 12½ per cent? What must he sell it for to make 25 per cent?

116. A man and his wife together eat a barrel of flour in 10 weeks; after using from it 4 weeks, it was found the man finished it in 8 weeks: how long would a barrel last the wife?

117. A grocer being asked the price of sugar, replied, 12½ cents a pound at retail, and $10 a hundred at wholesale: how much will a customer save on a hundredweight if he buys at wholesale? What per cent will he save?

118. When potatoes are retailed at $⅜ a half peck, and at $1½ a bushel, how much will a person save on 10 bushels, if he buys by the bushel?

119. A man divided his farm, containing 120 acres, between his two sons, giving the oldest ⅛ more than the youngest: how many acres had each?

120. Divide 84 into two such parts, that if ⅛ of the larger be taken from itself and added to the smaller, the parts will then be equal.

121. Two boys, A and B, have 140 pears; if A gives B ⅛ of his, each will have an equal number: how many has each?

122. A drover paid $225 for 75 sheep, and sold them at an advance of 20 per cent: how much did he make per head?

123. If you buy eggs at 3 cents for 4 eggs, and sell them at 3 for 4 cents, what per cent is the profit?

124. If 12 horses consume 6 tons of hay in 4 weeks, how many horses will 30 tons last 10 weeks?

125. A man being asked how much money he had, answered, if he had as much more, half as much more, and $2⅛, he should have $100: how many dollars had he?

126. If a market-man buys oranges at the rate of 3 cents apiece, and sells 3 for 7 cents, what per cent, and how much is his profit?

127. The distance from A to B is 60 miles, which is $\frac{1}{2}$ of $\frac{3}{4}$ the distance from B to C: what is the distance from A to C?

128. Divide 40 into two parts, which shall be to each other as 3 to 5.

129. Divide 30 into two parts, which shall be to each other as $\frac{2}{3}$ to $\frac{1}{10}$.

130. What sum must be put at 7 per cent to gain $84 interest in 1 year?

131. What sum must be put at 6 per cent to gain $54 interest in $1\frac{1}{2}$ year?

132. A young man having $200, loaned it at 8 per cent interest: how long will it take to double itself?

133. If you borrow $100, and in 8 years return $200 as principal and interest, what per cent do you pay?

134. If a man pays $12 annually for insuring his furniture worth $600, what is the rate per cent?

135. A lad bought a number of pears, and after eating a dozen of them, gave away 24, which was $\frac{2}{3}$ of the remainder: how many did he buy?

136. A man bequeathed 125 acres of land to his son and daughter, so that the number of acres each received was as 2 to 3: how many acres did each receive?

137. A farmer having rye and wheat worth 6s. and 10s. a bushel, wished to make a mixture worth 9 shillings: what proportion of each must he put in?

138. A grocer having two kinds of tea, worth 5s. and 7s. a pound, mixed 5 lbs. of each, and sold the mixture at 6s. 6d. a pound: how much did he make by the operation?

139. If a cistern has one pipe, which will fill it in 8 hours, and another which will empty it in 12 hours, how long will it take to fill it if both run together?

140. The sum of two fractions is $1\frac{8}{15}$, and the difference between them is $\frac{1}{3}$: what are the fractions?

141. What number is that, from which if 4 be subtracted, the remainder multiplied by 7, and 4 added to the product, the sum divided by 5, the quotient will be 12?

142. A lad being asked how many apples he had, replied, if you double the number, extract the square root, double the root, extract the cube root, triple the root, add 10, extract the square root, halve the root and multiply by 8, the square root of the product will be 4: how many apples had he?

143. A butcher bought a lot of turkeys and sheep, 30 in all, for $30; the turkeys were $3\frac{1}{2}$ apiece, and the sheep $2 apiece: how many of each did he buy?

144. What number is that, $\frac{1}{4}$ of which plus 6 is equal to $\frac{1}{3}$ of itself?

145. A company at a hotel spent £3 4s., which was as many shillings apiece as there were persons in the company: how many were there in the company?

146. A farmer exchanged a pasture 25 rods long and 9 rods wide, for one of equal area, in the form of a square: how many rods square was his new pasture?

147. Three men start from the same place at the same time, to travel round a circular field; one of them can travel the distance in 4 hours; another in 6 hours, and the third in 8 hours: in what time will they all meet at the starting place?

148. A cask has two pipes; one runs in at the rate of 29 gallons per hour, and the other runs out at the rate of 22 gallons per hour, and while both are open it is filled in 9 hours: how many gallons does it hold?

149. Two lads together caught 24 fish; one said to the other, give me 2 of your fish and I shall have twice as many as you; the other replied, if you give me 2, I shall have as many as you: how many had each?